The Derivative of $\ln(x)$ and More Chain Rule

The Derivative of $\ln(x)$

$$\frac{d}{dx}[\ln(x)] = \frac{1}{x}$$

Example 1: Find the derivative of $h(x) = 2\ln(x)$.

$$h'(x) = 2\left(\frac{1}{x}\right) = \frac{2}{x}$$

Example 2: Find the derivative of $y = \ln(x^2+5)$.

$f(u) = \ln(u)$ $g(x) = x^2+5$

$f'(u) = \frac{1}{u}$ $g'(x) = 2x$

$$y' = f'(g(x))g'(x) = \frac{1}{x^2+5}\cdot 2x = \boxed{\frac{2x}{x^2+5}}$$

Example 3: The position, in meters, of a particle moving on a straight line is given by $s(t) = (5t-2)^2\sqrt{3t}$, where t is measured in seconds. What is the velocity of the particle when $t = 3$?

Need product rule and chain rule!

$$v(t) = s'(t) = 2(5t-2)(5)\sqrt{3t} + (5t-2)^2\frac{3}{2\sqrt{3t}}$$

$$v(3) = (2)(13)(5)(3) + (13)^2\left(\frac{3}{6}\right)$$

$$= 390 + \frac{169}{2} = \boxed{\frac{949}{2}\ \text{m/s}}$$

Example 4: Find the derivative of $y = 3\cot^2(4x)$.

$Y = 3(\cot(4x))^2$ Need chain rule twice!

$f(u) = 3u^2$ $g(x) = \cot(4x)$

$f'(u) = 6u$

$Y' = f'(g(x))\,g'(x)$

$h(v) = \cot(v)$ $k(x) = 4x$

$h'(v) = -\csc^2(v)$ $k'(x) = 4$

$\Rightarrow g'(x) = h'(k(x))\,k'(x)$

$= -\csc^2(4x)\cdot 4$

$Y' = 6\cot(4x)(-\csc^2(4x))\cdot 4$

$= \boxed{-24\cot(4x)\csc^2(4x)}$

Example 5: Find the derivative of $y = e^{2x}\sin(7x)$.

Need product rule and chain rule.

$Y' = 2e^{2x}\sin(7x) + e^{2x}\cos(7x)\cdot 7$

$= \boxed{2e^{2x}\sin(7x) + 7e^{2x}\cos(7x)}$

DIY

1. Find the derivative of the following function.

$$y = \ln\left[\sqrt{(x^2+3)(x^4+3x^2+1)}\right]$$

$$Y = \frac{1}{2}\ln\left[(x^2+3)(x^4+3x^2+1)\right]$$

$$= \frac{1}{2}\left(\ln(x^2+3) + \ln(x^4+3x^2+1)\right)$$

$$= \frac{1}{2}\ln(x^2+3) + \frac{1}{2}\ln(x^4+3x^2+1)$$

$$Y' = \frac{1}{2}\cdot\frac{2x}{x^2+3} + \frac{1}{2}\cdot\frac{4x^3+6x}{x^4+3x^2+1}$$

$$\boxed{Y' = \frac{x}{x^2+3} + \frac{2x^3+3x}{x^4+3x^2+1}}$$