Higher Order Derivatives

When we take the derivative of a function $f(x)$, the result is another function, $f'(x)$. There's nothing stopping us from taking the derivative of the function $f'(x)$, or taking the derivative again, and again, and again, ...

Notation

Suppose $y = f(x)$.

- First Derivative: $f^{(1)}(x) = f'(x) = \frac{d}{dx}f(x) = y' = \frac{dy}{dx}$
- Second Derivative: $f^{(2)}(x) = f''(x) = \frac{d^2}{dx^2}f(x) = y'' = \frac{d^2y}{dx^2}$
- Third Derivative: $f^{(3)}(x) = f'''(x) = \frac{d^3}{dx^3}f(x) = y''' = \frac{d^3y}{dx^3}$
  $\vdots$
- n^{th} Derivative: $f^{(n)}(x) = \frac{d^n}{dx^n}f(x) = \frac{d^ny}{dx^n}$

Example 1: Find $f'''(x)$ for $f(x) = 2x^5 + 4x^3 + 3$.

Example 2: If $f^{(3)}(x) = 2e^{3x}\cos(2x)$, find $f^{(4)}(x)$.

Acceleration

Recall that the first derivative of position is velocity. The second derivative of position (so the first derivative of velocity), is *acceleration*.

Example 3: The position of an object, in feet, is given by $s(t) = \frac{13}{3}t^3 + 39t^2$, where t is measure in seconds. What is the acceleration of the object when its velocity is 2808 ft/s?

DIY

1. Find the second derivative of $h(x) = 3x^5 \ln(2x)$.

2. Find the second derivative of $g(x) = \frac{x^2}{x-2}$.