Higher Order Derivatives

When we take the derivative of a function $f(x)$, the result is another function, $f'(x)$. There's nothing stopping us from taking the derivative of the function $f'(x)$, or taking the derivative again, and again, and again, ...

Notation

Suppose $y = f(x)$.

- First Derivative: $f^{(1)}(x) = f'(x) = \frac{d}{dx}f(x) = y' = \frac{dy}{dx}$
- Second Derivative: $f^{(2)}(x) = f''(x) = \frac{d^2}{dx^2}f(x) = y'' = \frac{d^2y}{dx^2}$
- Third Derivative: $f^{(3)}(x) = f'''(x) = \frac{d^3}{dx^3}f(x) = y''' = \frac{d^3y}{dx^3}$

  $\vdots$
- n^{th} Derivative: $f^{(n)}(x) = \frac{d^n}{dx^n}f(x) = \frac{d^ny}{dx^n}$

Example 1: Find $f'''(x)$ for $f(x) = 2x^5 + 4x^3 + 3$.

$f'(x) = 10x^4 + 12x^2$

$f''(x) = 40x^3 + 24x$

$\boxed{f'''(x) = 120x^2 + 24}$

Example 2: If $f^{(3)}(x) = 2e^{3x}\cos(2x)$, find $f^{(4)}(x)$.

Need Product & chain rules!

$f^{(4)}(x) = 6e^{3x}\cos(2x) + 2e^{3x}(-\sin(2x))\cdot 2$

$= \boxed{6e^{3x}\cos(2x) - 4e^{3x}\sin(2x)}$

Acceleration

Recall that the first derivative of position is velocity. The second derivative of position (so the first derivative of velocity), is *acceleration.*

Example 3: The position of an object, in feet, is given by $s(t) = \frac{13}{3}t^3 + 39t^2$, where t is measure in seconds. What is the acceleration of the object when its velocity is 2808 ft/s?

$s'(t) = v(t) = 13t^2 + 78t$

$v(t) = 13t^2 + 78t = 2808$

$13t^2 + 78t - 2808 = 0$

$13(t^2 + 6t - 216) = 0$

$13(t-12)(t+18) = 0$

$t = 12$ $\quad t = -18$

time can't be negative

$s''(t) = v'(t) = a(t)$

$\Rightarrow a(t) = 26t + 78$

$a(12) = 26(12) + 78$

$= \boxed{390 \text{ ft}/s^2}$

DIY

1. Find the second derivative of $h(x) = 3x^5 \ln(2x)$.

Need product rule & chain rule.

$h'(x) = 15x^4 \ln(2x) + 3x^5\left(\frac{2}{2x}\right)$

$= 15x^4 \ln(2x) + 3x^4$

$h''(x) = 60x^3 \ln(2x) + 15x^4\left(\frac{2}{2x}\right) + 12x^3$

$= 60x^3 \ln(2x) + 15x^3 + 12x^3$

$= \boxed{60x^3 \ln(2x) + 27x^3}$

2. Find the second derivative of $g(x) = \frac{x^2}{x-2}$.

Quotient rule!

$$g'(x) = \frac{2x(x-2) - x^2(1)}{(x-2)^2} = \frac{2x^2 - 4x - x^2}{(x-2)^2}$$

$$= \frac{x^2 - 4x}{(x-2)^2}$$

$$g''(x) = \frac{(2x-4)(x-2)^2 - (x^2-4x)2(x-2)}{(x-2)^4}$$

$$= \frac{(2x-4)(x-2) - 2(x^2-4x)}{(x-2)^3}$$

$$= \frac{2x^2 - 8x + 8 - 2x^2 + 8x}{(x-2)^3}$$

$$= \boxed{\frac{8}{(x-2)^3}}$$