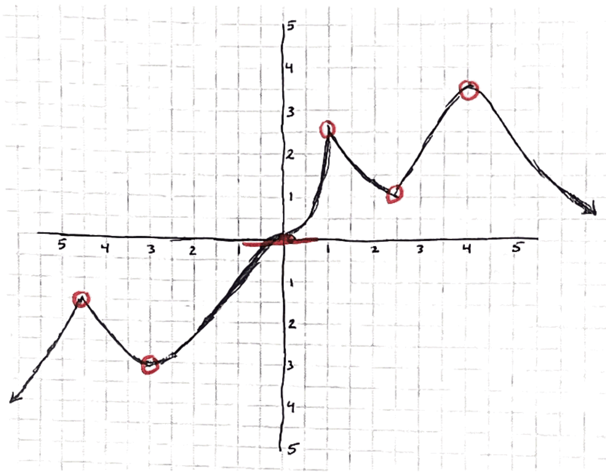
Relative Extrema and Critical Numbers

If $f(c) \geq f(x)$ for all x in a neighborhood about the point c, then $f(c)$ is a *relative maximum* ($f(c)$ is the highest point, of other points nearby, on the graph).

If $f(c) \leq f(x)$ for all x in a neighborhood about the point c, then $f(c)$ is a *relative minimum* ($f(c)$ is the lowest point, of other points nearby, on the graph).

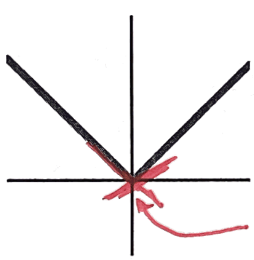
Example 1: List the relative maxima and minima from the following graph.



Relative Maxima: (-4.5, -1.5)
(1, 2.5)
(4, 3.5)

Relative Minima: (-3, -3)
(2.5, 1)

- Notice that maxima and minima occur where the derivative is zero or where it does not exist (as an example, if $f(x) = |x|$, then $f'(0) =$DNE).



Slope changes abruptly at $x=0$, so $f'(0) =$ DNE.

- Look at the origin in the graph in example 1. Notice that the derivative being zero at a point does not necessarily mean that there is a maximum or minimum at that point. Horizontal tangent line.

Critical Numbers

Let c be a point in the domain of f. If $f'(c) = 0$ or $f'(c) =$DNE, then c is a *critical number (point)* for the function f.

Example 2: Find the critical numbers of $f(x) = 2x^3 + 9x^2 + 12x$.

$f'(x) = 6x^2 + 18x + 12$

$6x^2 + 18x + 12 = 0$

$6(x^2 + 3x + 2) = 0$

$6(x+1)(x+2) = 0$

$\Rightarrow x = -1, \ x = -2$

$f'(x)$ is never undefined, so we only need to check where $f'(x) = 0$.

$x=-1$ and $x=-2$ are in the domain of f.

Critical numbers of f are $x=-1$ and $x=-2$.

Example 3: Find the critical numbers of $y = 3x^2 - \frac{4}{x^2}$.

$$y' = 6x + \frac{8}{x^3} = \frac{6x^4 + 8}{x^3}$$

$$\frac{6x^4 + 8}{x^3} = 0 \Rightarrow 6x^4 + 8 = 0 \Rightarrow x^4 = \frac{-4}{3}$$

No values of x make this equation true.

Notice that y' is undefined when $x=0$.
However, y is also undefined when $x=0$, so it is not a critical point.

So, y has no critical points.

Example 4: Find the critical numbers of $g(x) = 4x^3e^{3x}$.

$$g'(x) = 12x^2e^{3x} + 12x^3e^{3x}$$
$$= 12x^2e^{3x}(1+x)$$

$g'(x)$ is never undefined.

$$12x^2e^{3x}(1+x) = 0$$

$x=0$ never zero $x=-1$

$x=0$ and $x=-1$ are in the domain of g.

The critical numbers of g are $x=0$ and $x=-1$

DIY

1. Find the critical numbers of the following function.

$$f(x) = \frac{2x^2+1}{3x}$$

$$f'(x) = \frac{(4x)(3x)-(2x^2+1)(3)}{9x^2} = \frac{12x^2-6x^2-3}{9x^2}$$

$$= \frac{6x^2-3}{9x^2}$$

$$\frac{6x^2-3}{9x^2} = 0 \Rightarrow 6x^2-3=0$$
$$\Rightarrow 6x^2 = 3$$
$$\Rightarrow x^2 = 1/2$$
$$\Rightarrow x = \pm\sqrt{1/2}$$

in the domain of f.

f' is undefined at $x=0$, but $x=0$ is not in the domain of f.

The critical numbers are $x = \pm\sqrt{1/2}$