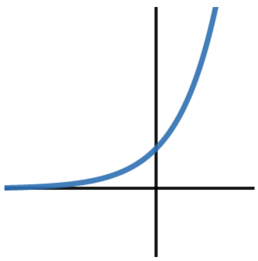
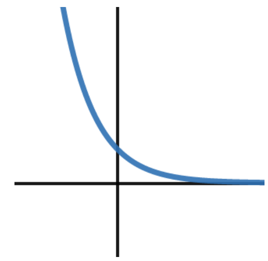
Increasing/Decreasing Functions

A function is *increasing* if $x > y$ implies $f(x) > f(y)$ (the function gets bigger as you move from left to right).

A function is *decreasing* if $x > y$ implies $f(x) < f(y)$ (the function gets smaller as you move from left to right).





Fact: If $f'(x) > 0$, then $f(x)$ is increasing. If $f'(x) < 0$, then $f(x)$ is decreasing.

Example 1: Find the intervals on which $f(x) = 2x^3 + 9x^2 + 12x$ is increasing and the intervals on which it is decreasing.

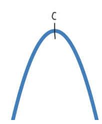
The First Derivative Test

Last time we saw that relative extrema of a function can occur at points where the derivative of the function is equal to zero.

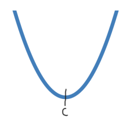
The First Derivative Test

Let c be a critical number of $f(x)$.

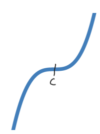
1. If $f'(x) > 0$ to the left of c and $f'(x) < 0$ to the right of c, then f has a **relative maximum** at c.



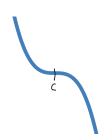
2. If $f'(x) < 0$ to the left of c and $f'(x) > 0$ to the right of c, then f has a **relative minimum** at c.



3. If $f'(x) > 0$ on both sides of c, then f has **neither** a relative maximum or minimum at c.



4. If $f'(x) < 0$ on both sides of c, then f has **neither** a relative maximum or minimum at c.



Example 1 revisited: Find the relative extrema of the function $f(x) = 2x^3 + 9x^2 + 12x$.

Example 2: Find the intervals on which $f(x) = 2x^4 - 3x^3$ is increasing, the intervals on which it is decreasing, and any relative extrema.

Example 3: If $g'(x) = e^{4x}\,(x^2 - 10)$, find the intervals on which $g(x)$ is increasing, the intervals on which it is decreasing, and the x-values at which it has relative extrema.

DIY

1. The critical numbers of $f(x) = \sin(x)$ on the interval of $(0, 2\pi)$ are $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$. Identify the x-values on $(0, 2\pi)$ at which $f(x)$ has a relative minimum.