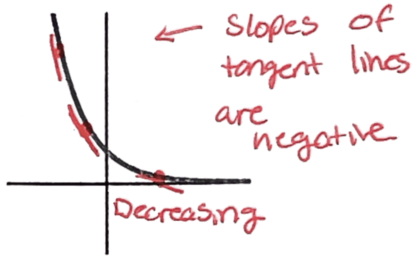
Increasing/Decreasing Functions

A function is *increasing* if $x > y$ implies $f(x) > f(y)$ (the function gets bigger as you move from left to right).

A function is *decreasing* if $x > y$ implies $f(x) < f(y)$ (the function gets smaller as you move from left to right).



Fact: If $f'(x) > 0$, then $f(x)$ is increasing. If $f'(x) < 0$, then $f(x)$ is decreasing.

Example 1: Find the intervals on which $f(x) = 2x^3 + 9x^2 + 12x$ is increasing and the intervals on which it is decreasing.

$$f'(x) = 6x^2 + 18x + 12$$
$$= 6(x^2 + 3x + 2)$$

$$6(x^2 + 3x + 2) = 0$$
$$6(x+1)(x+2) = 0$$
$$x = -1, \quad x = -2$$

Sign Chart: $f'(x)$ + 0 − 0 +

-3 -2 -1.5 -1 0

Inc: $(-\infty, -2) \cup (-1, \infty)$

Dec: $(-2, -1)$

The First Derivative Test

Last time we saw that relative extrema of a function can occur at points where the derivative of the function is equal to zero.

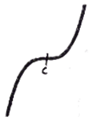
The First Derivative Test

Let c be a critical number of $f(x)$.

1. If $f'(x) > 0$ to the left of c and $f'(x) < 0$ to the right of c, then f has a **relative maximum** at c.

2. If $f'(x) < 0$ to the left of c and $f'(x) > 0$ to the right of c, then f has a **relative minimum** at c.

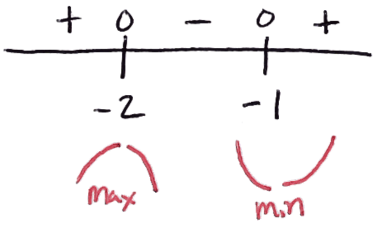
3. If $f'(x) > 0$ on both sides of c, then f has **neither** a relative maximum or minimum at c.



4. If $f'(x) < 0$ on both sides of c, then f has **neither** a relative maximum or minimum at c.

Example 1 revisited: Find the relative extrema of the function $f(x) = 2x^3 + 9x^2 + 12x$.

Sign Chart: $f'(x)$



(−2, −4) max

(−1, −5) min

Example 2: Find the intervals on which $f(x) = 2x^4 - 3x^3$ is increasing, the intervals on which it is decreasing, and any relative extrema.

$$f'(x) = 8x^3 - 9x^2 = x^2(8x-9)$$

$$x^2(8x-9) = 0 \Rightarrow x=0, \ x = 9/8$$

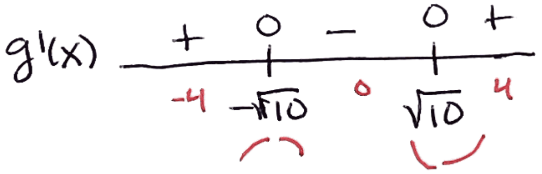
Inc: $(9/8, \infty)$

Dec: $(-\infty, 9/8)$

$\left(9/8, \frac{-2187}{2048}\right)$ min

Example 3: If $g'(x) = e^{4x}(x^2 - 10)$, find the intervals on which $g(x)$ is increasing, the intervals on which it is decreasing, and the x-values at which it has relative extrema.

$$\underset{\uparrow \text{ never zero}}{e^{4x}}(x^2-10) = 0 \Rightarrow x^2 = 10 \Rightarrow x = \pm\sqrt{10}$$



Inc: $(-\infty, -\sqrt{10}) \cup (\sqrt{10}, \infty)$

Dec: $(-\sqrt{10}, \sqrt{10})$

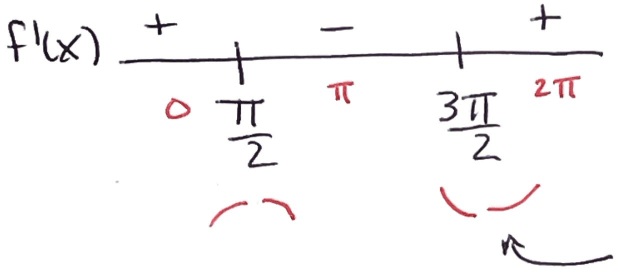
min at $x = \sqrt{10}$

max at $x = -\sqrt{10}$

DIY

1. The critical numbers of $f(x) = \sin(x)$ on the interval of $(0, 2\pi)$ are $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$. Identify the x-values on $(0, 2\pi)$ at which $f(x)$ has a relative minimum.

$f'(x) = \cos(x)$



Relative minimum at $x = \frac{3\pi}{2}$.