Absolute Extrema on an Interval

An *absolute maximum* is the highest point a function achieves on an interval.

An *absolute minimum* is the lowest point a function achieves on an interval.

Absolute Extrema on a Closed Interval

Fact: If $f(x)$ is continuous on a closed interval $[a, b]$, then $f(x)$ has *both* an absolute maximum and an absolute minimum on the interval.

Strategy for Finding Absolute Extrema on a Closed Interval

1. Find the critical numbers of the function.
2. Make a chart with a column for x-values and $f(x)$-values. In the x-values column, list the critical numbers and the endpoints of the interval.
3. Find the $f(x)$-values that correspond to the listed x-values.
4. Pick out the absolute maximum (point with the biggest $f(x)$ value) and the absolute minimum (point with the smallest $f(x)$ value).

Example 1: Find the absolute extrema of $f(x) = 3xe^{-x} + 7$ on the interval $[0, 5]$.

Absolute Extrema on Open and Half-Open, Half-Closed Intervals with One Critical Number

Fact: If a relative maximum occurs at the critical number, then this is the absolute maximum on the interval, but we cannot say anything about an absolute minimum.
If a relative minimum occurs at the critical point, then this is the absolute minimum on the interval, but we cannot say anything about an absolute maximum.
Use the first or second derivative tests to determine whether the critical number is a maximum or a minimum.

Example 2: Find the absolute maximum of $y = 3 - x^2$ on the interval $(-2, 2)$.

Example 3: Find the absolute minimum of $g(x) = 3x^3 + 4x^2 + 5$ on the interval $\left(\frac{-2}{9}, 3\right]$.

DIY

1. Find the absolute extrema of $y = 2x^3 + 9x^2 + 12x$ on the interval $\left[\frac{-3}{2}, 0\right]$.

2. Find the absolute maximum of $y = \frac{2x^2}{x+1}$ on the interval $(-3, -1)$.