Absolute Extrema on an Interval

An *absolute maximum* is the highest point a function achieves on an interval.

An *absolute minimum* is the lowest point a function achieves on an interval.

Absolute Extrema on a Closed Interval

Fact: If $f(x)$ is continuous on a closed interval $[a,b]$, then $f(x)$ has *both* an absolute maximum and an absolute minimum on the interval.

Strategy for Finding Absolute Extrema on a Closed Interval

1. Find the critical numbers of the function.
2. Make a chart with a column for x-values and $f(x)$-values. In the x-values column, list the critical numbers and the endpoints of the interval.
3. Find the $f(x)$-values that correspond to the listed x-values.
4. Pick out the absolute maximum (point with the biggest $f(x)$ value) and the absolute minimum (point with the smallest $f(x)$ value).

Example 1: Find the absolute extrema of $f(x) = 3xe^{-x} + 7$ on the interval $[0, 5]$.

$f'(x) = 3e^{-x} - 3xe^{-x}$

$= 3e^{-x}(1-x)$

$3e^{-x}(1-x) = 0$

$3e^{-x}$: never zero; $1-x$: $x=1$

x	$f(x)$	
0	7	min
1	$3e^{-1}+7 \approx 8.104$	max
5	$15e^{-5}+7 \approx 7.101$	

$\boxed{\text{Abs. max } (1, 3e^{-1}+7) \quad \text{Abs. min } (0, 7)}$

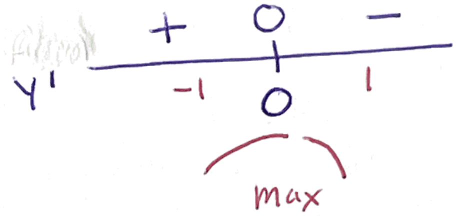
Absolute Extrema on Open and Half-Open, Half-Closed Intervals with One Critical Number

Fact: If a relative maximum occurs at the critical number, then this is the absolute maximum on the interval, but we cannot say anything about an absolute minimum.
If a relative minimum occurs at the critical point, then this is the absolute minimum on the interval, but we cannot say anything about an absolute maximum.
Use the first or second derivative tests to determine whether the critical number is a maximum or a minimum.

Example 2: Find the absolute maximum of $y = 3 - x^2$ on the interval $(-2, 2)$.

$y' = -2x$

$-2x = 0 \Rightarrow x = 0$



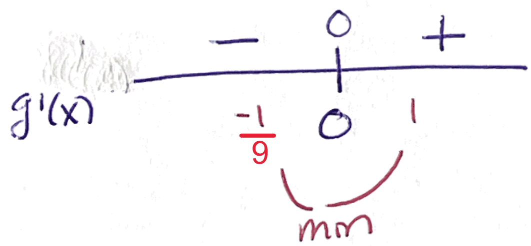
$\boxed{\text{Absolute max at } (0,3)}$

Example 3: Find the absolute minimum of $g(x) = 3x^3 + 4x^2 + 5$ on the interval $\left(\frac{-2}{9}, 3\right]$.

$g'(x) = 9x^2 + 8x$
$= x(9x+8)$

$x(9x+8) = 0$

$x = 0$ $x = -8/9$ not in the interval



$\boxed{\text{Absolute min at } (0,5)}$

DIY

1. Find the absolute extrema of $y = 2x^3 + 9x^2 + 12x$ on the interval $\left[\frac{-3}{2}, 0\right]$.

$y' = 6x^2 + 18x + 12$
$= 6(x^2 + 3x + 2)$
$= 6(x+1)(x+2)$

$6(x+1)(x+2) = 0$

$x = -1$ $x = -2$ not in interval

x	$f(x)$	
$-3/2$	$-9/2$	
-1	-5	min
0	0	max

Abs. min at $(-1,-5)$
Abs. max at $(0,0)$

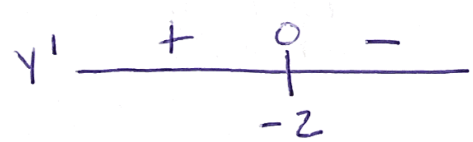
2. Find the absolute maximum of $y = \frac{2x^2}{x+1}$ on the interval $(-3, -1)$.

$$y' = \frac{4x(x+1) - 2x^2}{(x+1)^2} = \frac{2x^2 + 4x}{(x+1)^2}$$

Denominator zero at $x=-1$, but this is not in the domain of y.

$\frac{2x^2+4x}{(x+1)^2} = 0 \Rightarrow 2x^2 + 4x = 0$
$\Rightarrow 2x(x+2) = 0$

not in interval $\rightarrow x = 0$, $x = -2$



$\boxed{\text{Abs. max at } (-2,-8)}$