Rules of Exponentials and Logarithms

Let a, m, and n be real numbers.

- $a^m \cdot a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^{-m} = \frac{1}{a^m}$
- $a^0 = 1$, if $a \neq 0$

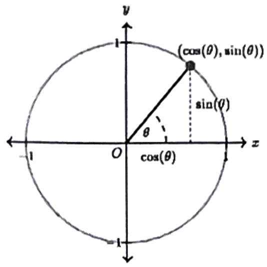
The following rules hold for any $\log_c(x)$, $c > 0$, but are presented using the *natural log* function $\log_e(x) = \ln(x)$, as we will use this most often. Let a and b be real numbers.

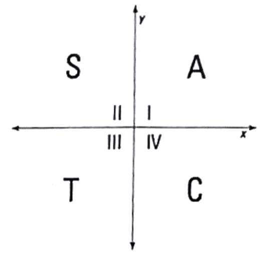
- $\ln(ab) = \ln(a) + \ln(b)$
- $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$
- $\ln\left(a^b\right) = b\ln(a)$
- $\ln(e) = 1$ (In general, $\log_c(c) = 1$)
- $\ln(1) = 0$ (In general, $\log_c(1) = 0$)
- $\ln(x)$ is undefined for $x \leq 0$. (In general, $\log_c(x)$ is undefined for $x \leq 0$.)

A Brief Trigonometry Review

θ	$\sin(\theta)$	$\cos(\theta)$
0	0	1
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1	0
π	0	-1
$\frac{3\pi}{2}$	-1	0
2π	0	1

At any point on the unit circle the radius (hypotenuse of the right triangle) is always 1, the x-coordinate corresponds to the cosine function (at an appropriate value of θ), and the y-coordinate corresponds to the sine function (at an appropriate value of θ).





Where are the trig functions positive? All trig functions are positive in the first quadrant. Sine is positive in the second quadrant, tangent is positive in the third quadrant, and cosine is positive in the fourth quadrant. A helpful way to remember this is, "All Students Take Calculus," (ASTC).

Some Trig Identities

- $\sec\theta = \frac{1}{\cos\theta}$
- $\csc\theta = \frac{1}{\sin\theta}$
- $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1}{\cot\theta}$
- $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$
- $\sin^2\theta + \cos^2\theta = 1$

DIY

1. Use rules of exponents to simplify the following expression.

$$(e^x)^6\,(e^{-x})^3$$

$$= e^{6x}e^{-3x} = e^{6x-3x} = \boxed{e^{3x}}$$

2. Use rules of exponents and logarithms to simplify the following expression.

$$e^{7+\ln(3x)}$$

$$= e^7 e^{\ln(3x)} = e^7(3x) = \boxed{3xe^7}$$

3. Solve the following equations for x. Keep your answers exact.

$$\ln\left(x^4\right) = 12 \qquad \text{and} \qquad \ln\left(x^3\right) = 27$$

$$\Rightarrow e^{\ln(x^4)} = e^{12}$$
$$\Rightarrow x^4 = e^{12}$$
$$\Rightarrow x = \pm\sqrt[4]{e^{12}} = \pm\left(e^{12}\right)^{1/4} = \pm e^3$$
$$\boxed{x = \pm e^3}$$

$$\Rightarrow e^{\ln(x^3)} = e^{27}$$
$$\Rightarrow x^3 = e^{27}$$
$$\Rightarrow x = \sqrt[3]{e^{27}} = \left(e^{27}\right)^{1/3} = e^9$$
$$\boxed{x = e^9}$$

4. Use rules of logarithms to rewrite the following expression as sums, differences, and/or multiples of logarithms. Do not leave any negative exponents.

$$\ln\left(\frac{u^2 v}{w}\right)$$

$$= \ln(u^2 v) - \ln(w)$$
$$= \ln(u^2) + \ln(v) - \ln(w)$$
$$= 2\ln(u) + \ln(v) - \ln(w)$$

5. Use the given information to find the exact values of the remaining trigonometric functions of θ. $\sin(\theta) = \frac{3}{4}$ in Quadrant II.

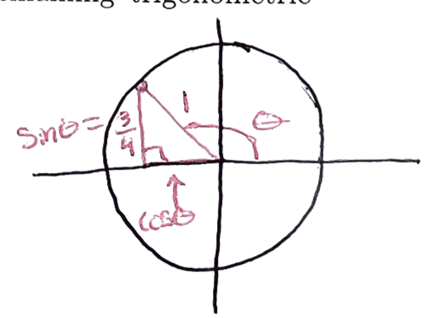
$\cos(\theta) = \dfrac{-\sqrt{7}}{4}$

$\tan(\theta) = \dfrac{\sin\theta}{\cos\theta} = \dfrac{3/4}{-\sqrt{7}/4} = \dfrac{-3}{\sqrt{7}}$

$\csc(\theta) = \dfrac{1}{\sin\theta} = \dfrac{4}{3}$

$\sec(\theta) = \dfrac{1}{\cos\theta} = -4/\sqrt{7}$

$\cot(\theta) = \dfrac{1}{\tan\theta} = -\dfrac{\sqrt{7}}{3}$



Pythagorean Theorem:

$$\left(\tfrac{3}{4}\right)^2 + \cos^2\theta = 1$$
$$\Rightarrow \cos\theta = \pm\sqrt{1 - 9/16}$$
$$\Rightarrow \cos\theta = \pm\sqrt{7/16}$$

Since we are in the second quadrant, $\cos\theta$ must be negative. Thus, $\cos\theta = -\sqrt{7/16}$