Antiderivatives and Indefinite Integration I

Antiderivatives (integrals) "undo" derivatives. When taking the antiderivative, think of doing the opposite of taking the derivative.

Can we find a function $F(x)$ so that $F'(x) = 5x^2$?

- We need to do the opposite of taking the derivative: add one to the exponent and then divide!

$$F'(x) = 5x^2 \Rightarrow F(x) = \frac{5}{3}x^3$$

- Check: $F(x) = 5/3\, x^3 \Rightarrow F'(x) = 5x^2$ ✓

- What about $F(x) = 5/3\, x^3 + 10$?

$$F'(x) = 5x^2$$

- Hmm: We say that the antiderivative of $5x^2$ is $5/3\, x^3 + C$, where C is a constant.

Example 1: Evaluate the following integral.

$$\int \frac{x^2+5}{4}\,dx$$

$$= \int \frac{1}{4}(x^2+5)\,dx = \frac{1}{4}\int (x^2+5)\,dx$$

$$= \frac{1}{4}\left(\frac{1}{3}x^3 + 5x\right) + C$$

$$= \boxed{\frac{1}{12}x^3 + \frac{5}{4}x + C}$$

Example 2: Evaluate the following integral.

$$\int \frac{x^2 - x^5}{\sqrt{x}}\, dx$$

$$= \int \left(\frac{x^2}{\sqrt{x}} - \frac{x^5}{\sqrt{x}} \right) dx = \int \left(\frac{x^2}{x^{1/2}} - \frac{x^5}{x^{1/2}} \right) dx$$

$$= \int (x^{3/2} - x^{9/2})\, dx = \boxed{\frac{2}{5} x^{5/2} - \frac{2}{11} x^{11/2} + C}$$

Example 3: Evaluate the following integral.

$$\int \left(4\cot(x)\sin(x) + 3\sec^2(x)\right)\, dx$$

$$= \int \left(4 \frac{\cos x}{\sin x} \sin x + 3\sec^2 x\right) dx = \int (4\cos x + 3\sec^2 x)\, dx$$

$$= \boxed{4\sin(x) + 3\tan(x) + C}$$

DIY

1. Evaluate the following integral.

$$\int \frac{2 + 3xe^x}{x}\, dx$$

$$= \int \left(\frac{2}{x} + \frac{3xe^x}{x} \right) dx = 2\int \frac{1}{x}\, dx + 3\int e^x\, dx$$

$$= \boxed{2\ln|x| + 3e^x + C}$$

use absolute value because we can't plug negative numbers into ln(x).

Basic Differentiation Rules	Basic Integration Rules
$\frac{d}{dx}C = 0$	$\int 0\ dx = C$
$\frac{d}{dx}(kx) = k$	$\int k\ dx = kx + C$
$\frac{d}{dx}[kf(x)] = kf'(x)$	$\int [kf(x)]\ dx = k \int f(x)\ dx$
$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$	$\int [f(x) \pm g(x)]\ dx = \int f(x)\ dx \pm \int g(x)\ dx$
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n\ dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$ (Power Rule)
$\frac{d}{dx}\sin x = \cos x$	$\int \cos x\ dx = \sin x + C$
$\frac{d}{dx}\cos x = -\sin x$	$\int \sin x\ dx = -\cos x + C$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \sec^2 x\ dx = \tan x + C$
$\frac{d}{dx}\cot x = -\csc^2 x$	$\int \csc^2 x\ dx = -\cot x + C$
$\frac{d}{dx}\sec x = \sec x \tan x$	$\int \sec x \tan x\ dx = \sec x + C$
$\frac{d}{dx}\csc x = -\csc x \cot x$	$\int \csc x \cot x\ dx = -\csc x + C$
$\frac{d}{dx}e^x = e^x$	$\int e^x\ dx = e^x + C$
$\frac{d}{dx}\ln x = \frac{1}{x}, x > 0$	$\int \frac{1}{x}\ dx = \ln\lvert x\rvert + C$