Antiderivatives and Indefinite Integration II

Today we are going to solve *initial value problems.* We will be given information to solve for the constant of integration, C.

Example 1: Given that $y' = \frac{5}{x}$ and $y(e) = 3$, find $y(e^3)$.

$$Y = \int \frac{5}{x}\,dx = 5\ln|x| + C$$

$$Y(e) = 3 = 5\ln(e) + C \Rightarrow 3 = 5 + C \Rightarrow C = -2$$

$$\Rightarrow Y = 5\ln|x| - 2$$

$$Y(e^3) = 5\ln(e^3) - 2 = 5 \cdot 3\ln(e) - 2 = 15 - 2 = 13$$

Example 2: Given that $y'' = 2e^x + 4$, $y'(0) = 1$, and $y(3) = -1$, find $y(2)$.

$$Y' = \int (2e^x + 4)\,dx = 2e^x + 4x + C$$

$$Y'(0) = 1 = 2 + C \Rightarrow C = -1$$

$$\Rightarrow Y' = 2e^x + 4x - 1$$

$$Y = \int (2e^x + 4x - 1)\,dx = 2e^x + 2x^2 - x + D$$

$$Y(3) = -1 = 2e^3 + 18 - 3 + D \Rightarrow D = -16 - 2e^3$$

$$\Rightarrow Y = 2e^x + 2x^2 - x - 16 - 2e^3$$

$$Y(2) = 2e^2 + 8 - 2 - 16 - 2e^3$$

$$= \boxed{2e^2 - 2e^3 - 10}$$

Example 3: The rate of growth, $\frac{dP}{dt}$, of a population of bacteria is proportional to the square root of t with a constant coefficient of 3. If the initial population is 200, approximate the population after 10 days.

$P' = \frac{dP}{dt} = 3\sqrt{t}$, $P(0) = 200$, find $P(10)$.

$P = \int 3\sqrt{t}\,dt = \int 3t^{1/2}\,dt = 2t^{3/2} + C$

$P(0) = 200 = 0 + C \Rightarrow C = 200$

$\Rightarrow P(t) = 2t^{3/2} + 200$

$P(10) = 2(10)^{3/2} + 200 \approx \boxed{263}$

Example 4: A hot air balloon is rising vertically with a velocity of 4 ft/s. A ball is released from the hot air balloon when it is 80 ft above the ground. Use $a(t) = -32$ ft/s as acceleration due to gravity.

(a) How long will it take the ball to reach the ground?

(b) At what velocity will it hit the ground?

$a(t) = -32$ ft/s, $v(0) = 4$, $S(0) = 80$.

$v(t) = \int -32\,dt = -32t + C$, $v(0) = 4 = -32(0) + C \Rightarrow C = 4$

$\Rightarrow v(t) = -32t + 4$

$S(t) = \int(-32t + 4)\,dt = -16t^2 + 4t + D$

$S(0) = 80 = -16(0)^2 + 4(0) + D \Rightarrow D = 80$

$\Rightarrow S(t) = -16t^2 + 4t + 80$

(a) $-16t^2 + 4t + 80 = 0$

quadratic formula:

$t = \frac{1}{8}(1 - \sqrt{321})$ and $t = \frac{1}{8}(1 + \sqrt{321})$

negative positive

$\boxed{t = \frac{1}{8}(1 + \sqrt{321}) \approx 2.36\text{ s}}$

(b)

$v\left(\frac{1}{8}(1 + \sqrt{321})\right) \approx \boxed{-71.67\text{ ft/s}}$

DIY

1. Given that $y' = 2 - 3\sin(x)$ and $y(0) = 3$, find y.

$$y = \int (2 - 3\sin(x))\,dx = 2x + 3\cos(x) + C$$

$$y(0) = 3 = 2(0) + 3\cos(0) + C$$

$$3 = 3 + C \Rightarrow \underline{C = 0}$$

$$\Rightarrow \boxed{y = 2x + 3\cos(x)}$$