Area and Riemann Sums

Sigma Notation

The sigma symbol, $\sum$, means to add (sum) things up.

Example 1: Evaluate the following sum.

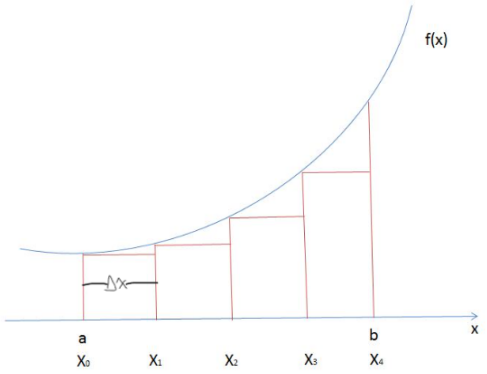
$$\sum_{i=1}^{4} (2i + 3)$$

Example 2: Rewrite the following sum using sigma notation.

$$\sqrt{3-2} + \sqrt{4-2} + \sqrt{5-2} + \sqrt{6-2} + \cdots + \sqrt{n-2}$$

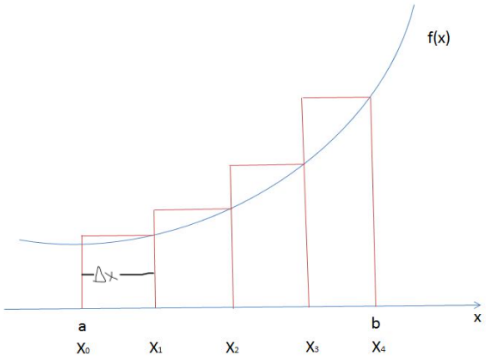
Area and Riemann Sums

Left and Right Riemann Sums use rectangles to estimate the area between a curve and the x-axis. If we want to use Riemann sums to approximate the area between a curve and the x-axis on the interval $[a, b]$ with n rectangles, then we partition the x-axis between a and b into n subintervals of width $\Delta x = \frac{b-a}{n}$. When using the *Left Riemann Sum*, L_n, we create rectangles from the left endpoint of each subinterval. When using the *Right Riemann Sum*, R_n, we create rectangles from the right endpoint of each subinterval. Examples using 4 rectangles are pictured below.



Left Riemann Sum (form rectangles at the left endpoints)

$$L_4 = \sum_{i=0}^{3} f(x_i)\Delta x$$



Right Riemann Sum (form rectangles at the right endpoints)

$$R_4 = \sum_{i=1}^{4} f(x_i)\Delta x$$

Example 3: Use the left and right Riemann sums with 3 rectangles to estimate the (signed) area under the curve of $y = 2x^3$ on the interval $[0, 6]$.

General Riemann Sum Formulas

In general, the formulas for the left and right Riemann sums for $f(x)$ on $[a, b]$ using n rectangles are:

$$L_n = \sum_{i=0}^{n-1} f\left(i\Delta x + a\right)\Delta x$$

and

$$R_n = \sum_{i=1}^{n} f\left(i\Delta x + a\right)\Delta x.$$

Example 4: Use the left and right Riemann sums with 90 rectangles to estimate the signed area under the curve of $y = 2x^5 + 3$ on the interval $[10, 20]$.

DIY

1. Evaluate the following sum.

$$\sum_{i=2}^{5} \left(i^2 + 1\right)$$

2. Use sigma notation to rewrite the following sum.

$$\left(2(0)^5 + 1\right) + \left(2(1)^5 + 1\right) + \left(2(2)^5 + 1\right) + \cdots + \left(2(n)^5 + 1\right)$$