Definite Integrals I

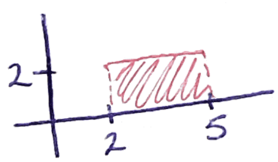
We have seen how to approximate the (signed) area under a curve using left and right Riemann sums. As we increase the number of rectangles used the estimate gets closer to the actual area. If we let the number of rectangles approach infinity (take the limit as $n \to \infty$), then we get the **exact** area between the curve and the x-axis. We denote the exact area between the curve and the x-axis using a *definite integral*:

$$\int_a^b f(x)\,dx.$$

This definite integral is equal to the area between the curve $f(x)$ and the x-axis on the interval $[a, b]$. We call a the *lower limit of integration*, b the *upper limit of integration*, and $f(x)$ the *integrand*. The dx tells us to take the integral (antiderivative) with respect to the variable x.

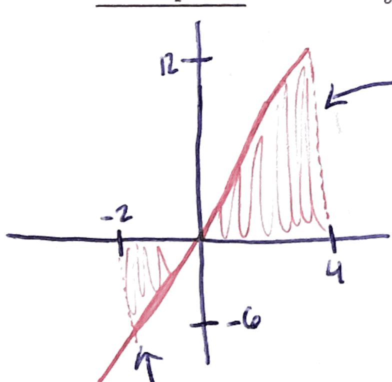
Geometric Interpretation: Definite Integral of a function = area between the graph of the function and the x-axis.

Example 1: Evaluate $\int_2^5 2\,dx$ by using geometric formulas.



$$A = 2(5-2)$$
$$= 2(3)$$
$$= 6$$

Example 2: Evaluate $\int_{-2}^4 3x\,dx$ by using geometric formulas.



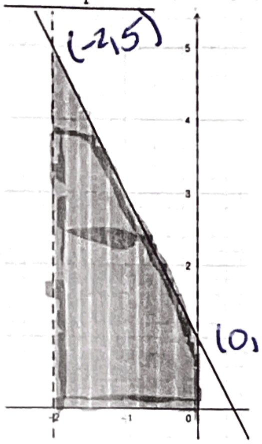
$$A = \left(\tfrac{1}{2}\right)(4)(12)$$
$$= 24$$

$$\text{Total area} = 24 - 6$$
$$= 18$$

$$A = 2(6)\left(\tfrac{1}{2}\right)$$
$$= 6 \Rightarrow -6$$

below x-axis

Example 3: Write an integral that represents the shaded area.



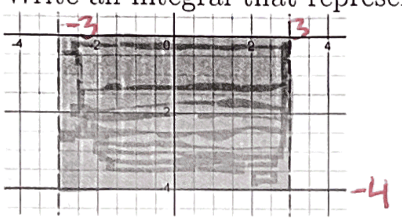
Slope: $\frac{5-1}{-2-0} = \frac{4}{-2} = -2$

$y = -2x+1$

$$A = \int_{-2}^{0} (-2x+1)\,dx$$

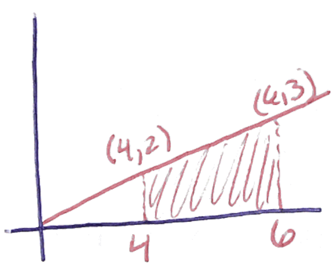
DIY

1. Write an integral that represents the shaded area.



$$A = \int_{-3}^{3} -4\,dx$$

2. Evaluate $\int_4^6 \frac{1}{2}x\,dx$ by using geometric formulas. Recall that the area of a trapezoid is $A = \frac{1}{2}(\text{base } 1 + \text{base } 2) \cdot (\text{height})$.



$$A = \frac{1}{2}(2+3)(2)$$
$$= 5$$