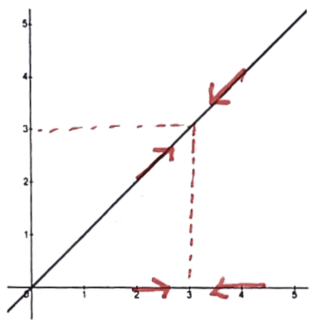
Finding Limits Numerically

The *limit* of a function is the value a function approaches as x approaches a particular value. If $f(x)$ approaches L as x approaches c, we say the limit of $f(x)$ as x approaches c is L, and we write $\lim_{x\to c} f(x) = L$. Think: As the x-values get closer to c, the y-values ($y = f(x)$) get closer to L.

Example 1: If $f(x) = x$ and $c = 3$, find $\lim_{x\to c} f(x)$.



$$\lim_{x\to 3} x = 3$$

Limits Come in Four Flavors

1. L: a finite value
2. ∞: The function gets bigger and bigger as x approaches c.
3. $-\infty$: The function gets smaller and smaller as x approaches c.
4. Does Not Exist (DNE): The function doesn't approach a specific value as x approaches c.

We can estimate the limit of a function by evaluating the function at numbers close to c.

Example 2: Find the following limit numerically.

$$\lim_{x\to 0} \frac{6x}{x^2+3x} = 2$$

x	-0.01	-0.001	-0.0001	0	0.0001	0.001	0.01
$f(x)$	2.0067	2.0007	2.0001	–	1.9999	1.9993	1.9934

Notice that $f(x)$ need not be defined at the point c in order to find the limit!

Example 3: Find the following limit numerically.

$$\lim_{x\to -3} \frac{7}{(x+3)^2} = \infty$$

x	-3.01	-3.001	-3.0001	-3	-2.9999	-2.999	-2.99
$f(x)$	70,000	7,000,000	700,000,000	–	700,000,000	7,000,000	70,000

One-Sided Limits

- Left-Sided Limit: $\lim_{x\to c^-} f(x)$; Only look at values of x that are less than (to the left of) c.
- Right-Sided Limit: $\lim_{x\to c^+} f(x)$; Only look at values of x that are greater than (to the right of) c.

Be careful to notice the difference between limits at negative numbers and left-sided limits. $\lim_{x\to -c} f(x)$ is generally not the same as $\lim_{x\to c^-} f(x)$.

Example 4: Find the following limits numerically.

$$\lim_{x\to 2^-} \frac{3}{x-2} = -\infty \qquad \lim_{x\to 2^+} \frac{3}{x-2} = \infty \qquad \lim_{x\to 2} \frac{3}{x-2} = \text{DNE}$$

x	1.99	1.999	1.9999	2	2.0001	2.001	2.01
$f(x)$	-300	-3000	-30,000	–	30,000	3,000	300

If you are only asked to find one of the one-sided limits, you only need to create the appropriate half of the chart.

Example 5: Find the following limits numerically.

$$\lim_{x\to 0^-} f(x) = 0 \qquad \lim_{x\to 0^+} f(x) = 0 \qquad \lim_{x\to 0} f(x) = 0$$

where

$$f(x) = \begin{cases} 3\sin(x) & x < 0 \\ 2x & x \geq 0 \end{cases}$$

Make sure your calculator is in radians when working with trig functions in this course.

x	-0.01	-0.001	-0.0001	0	0.0001	0.001	0.01
$f(x)$	-0.0300	-0.0030	-0.0003	–	0.0002	0.002	0.02

Fact

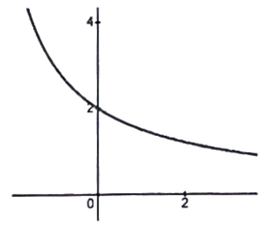
$\lim_{x\to c} f(x) = L$ if and only if $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = L$.
* Here we allow $L = \pm\infty$.

Finding Limits Graphically

We can also determine the limit of a function by looking at its graph.

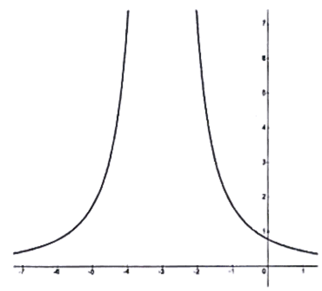
Example 2 Revisited:

$$\lim_{x \to 0} \frac{6x}{x^2 + 3x} = 2$$



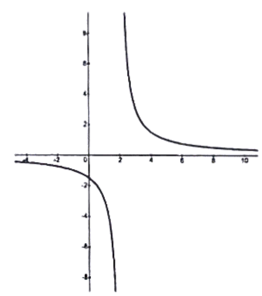
Example 3 Revisited:

$$\lim_{x \to -3} \frac{7}{(x+3)^2} = \infty$$



Example 4 Revisited:

$$\lim_{x \to 2} \frac{3}{x-2} = \text{DNE}$$



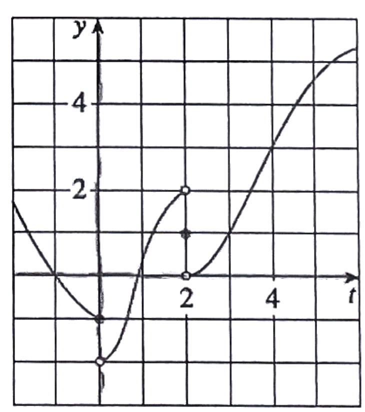
Example 6: Find the following limits and function values graphically.

$$\lim_{t \to 2^-} f(t) = 2 \qquad \lim_{t \to 4^-} f(t) = 3$$

$$\lim_{t \to 2^+} f(t) = 0 \qquad \lim_{t \to 4^+} f(t) = 3$$

$$\lim_{t \to 2} f(t) = \text{DNE} \qquad \lim_{t \to 4} f(t) = 3$$

$$f(2) = 1 \qquad f(4) = 3$$



Example 7: Find the following limits and function values graphically.

$\lim_{x\to -3^-} f(x) = -\infty$

$\lim_{x\to -3^+} f(x) = \infty$

$\lim_{x\to -3} f(x) = \text{DNE}$

$f(-3) = \text{Undefined}$

$\lim_{x\to 2^-} f(x) = -\infty$

$\lim_{x\to 2^+} f(x) = -\infty$

$\lim_{x\to 2} f(x) = -\infty$

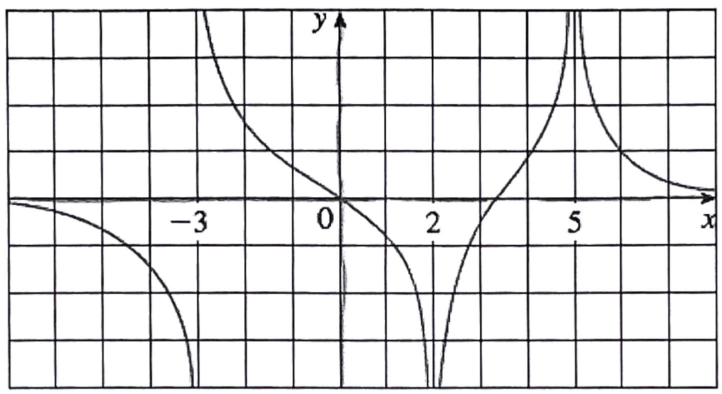
$f(2) = \text{Undefined}$

$\lim_{x\to 5^-} f(x) = \infty$

$\lim_{x\to 5^+} f(x) = \infty$

$\lim_{x\to 5} f(x) = \infty$

$f(5) = \text{Undefined}$



DIY

1. Find the following limits and function values graphically.

$\lim_{x\to -2^-} f(x) = 1$

$\lim_{x\to -2^+} f(x) = 1$

$\lim_{x\to -2} f(x) = 1$

$f(-2) = 1$

$\lim_{x\to 1^-} f(x) = 1$

$\lim_{x\to 1^+} f(x) = 2$

$\lim_{x\to 1} f(x) = \text{DNE}$

$f(1) = -2$