Finding Limits Analytically

We will examine three different cases.
When asked to find $\lim_{x\to c} f(x)$ analytically, **always** look at $f(c)$ first!

Case I

$f(c) =$ a number (even 0 is okay). That number is the answer to the limit.

Example 1: Evaluate the following limit analytically.

$$\lim_{x\to 8}(3x - 12)$$

Case II

$f(c) = \frac{\text{nonzero number}}{0}$. Case II tells us that $f(x)$ has a *vertical asymptote* at $x = c$. At a vertical asymptote, the value of the limit can be $+\infty$, $-\infty$, or DNE. We have to check right an left hand limits to decide which it is.

Example 2: Evaluate the following limit analytically.

$$\lim_{x\to -3} \frac{7}{(x+3)^2}$$

Example 3: Evaluate the following limit analytically.

$$\lim_{x\to 2} \frac{3}{x-2}$$

Case III

$f(c) = \frac{0}{0}$. In this case we need to do some algebra on $f(x)$ to end up with a "new" limit that falls into Case I or Case II.

Example 4: Evaluate the following limit analytically.

$$\lim_{x\to 0} \frac{x^2-4x}{x^2+2x}$$

Example 5: Evaluate the following limit analytically.

$$\lim_{x \to 1} \frac{x^2 - x}{(x-1)^2}$$

We can also find limits of *piecewise functions* analytically.

Example 6: Evaluate the following limits analytically, where

$$f(x) = \begin{cases} 3x^2 + 2, & x \leq 0 \\ 8x + 2, & 0 < x < 1 \\ -8x + 2, & x \geq 1 \end{cases}$$

$$\lim_{x \to 0^-} f(x) = \qquad \lim_{x \to 1^-} f(x) =$$

$$\lim_{x \to 0^+} f(x) = \qquad \lim_{x \to 1^+} f(x) =$$

$$\lim_{x \to 0} f(x) = \qquad \lim_{x \to 1} f(x) =$$

Properties of Limits

Let c, k, L, and K be real numbers and n a positive integer. If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = K$, then we have the following.

- $\lim_{x \to c} [kf(x)] = kL$
- $\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$
- $\lim_{x \to c} [f(x)g(x)] = LK$
- $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}$, assuming $K \neq 0$
- $\lim_{x \to c} [f(x)]^n = L^n$

DIY

1. Find the following limit analytically.

$$\lim_{x \to 0} \frac{x^3 + 2x^2}{x^2 - 3x}$$

2. Find the following limit analytically.

$$\lim_{x \to 0} \frac{x^2 - 3x + 2}{x^2 + 3x + 2}$$

3. Find the following limit analytically.

$$\lim_{x \to 6} \frac{-1}{x - 6}$$