# Definite Integrals II

## Properties of Definite Integrals

Let $a, b, c, k$ be constants.

- $\int_a^a f(x)\,dx = 0$
- $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
- $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$
- $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$
- $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

- Indefinite Integrals: Final answer is a function, in terms of the variable, plus a constant $C$.
- Definite Integrals: Final answer is a number (don't need $+C$).

Example 1: Given that $\int_1^2 x^3\,dx = \frac{15}{4}$, $\int_1^2 x^2\,dx = \frac{7}{3}$, and $\int_1^2 1\,dx = 1$, compute the following integral.

$$\int_1^2 \left(2x^3 - 4x^2 + 5\right)\,dx$$

$$= 2\int_1^2 x^3 dx - 4\int_1^2 x^2\,dx + 5\int_1^2 1\,dx$$

$$= 2(15/4) - 4(7/3) + 5(1) = \frac{19}{6}$$

Example 2: Given that $\int_{-3}^4 5x\,dx = \frac{35}{2}$, compute the following integrals.

$$\int_4^{-3} 5x\,dx \qquad \text{and} \qquad \int_{-3}^4 10x\,dx$$

$$= -\int_{-3}^4 5x\,dx$$

$$= -\frac{35}{2}$$

$$= 2\int_{-3}^4 5x\,dx$$

$$= 35$$

Example 3: Given that $\int_{-10}^{20} g(t)\,dt = 50$ and $\int_{15}^{20} g(t)\,dt = 72$, find $\int_{-10}^{15} g(t)\,dt$.

$$\int_{-10}^{20} g(t)\,dt = \int_{-10}^{15} g(t)\,dt + \int_{15}^{20} g(t)\,dt$$

$$50 = \int_{-10}^{15} g(t)\,dt + 72$$

$$\Rightarrow \quad -22 = \int_{-10}^{15} g(t)\,dt$$

## DIY

1. Given that $\int_0^5 f(x)\,dx = 7$, $\int_5^6 f(x)\,dx = 3$, and $\int_0^5 g(x)\,dx = 9$, find the following integrals.

   (a) $\int_0^6 f(x)\,dx = \int_0^5 f(x)dx + \int_5^6 f(x)dx = 7+3 = 10$

   (b) $\int_6^5 f(x)\,dx = -\int_5^6 f(x)dx = -3$

   (c) $\int_0^5 3f(x)\,dx = 3\int_0^5 f(x)dx = 3(7) = 21$

   (d) $\int_0^5 (3f(x) + 2g(x))\,dx$

   $= 3\int_0^5 f(x)dx + 2\int_0^5 g(x)dx$

   $= 3(7) + 2(9) = 21 + 18 = 39$

2. Given that $\int_a^b 13f(x)\,dx = 3$, find $\int_a^b 7f(x)\,dx$.

$$\int_a^b 7f(x)dx = 7\int_a^b f(x)dx = 7\left(\frac{3}{13}\right) = \frac{21}{13}$$

$$\int_a^b 13f(x)dx = 3 \Rightarrow 13\int_a^b f(x)dx = 3 \Rightarrow \int_a^b f(x)dx = \frac{3}{13}$$

3. Given that $\int_a^b h(t)\,dt = 2$, find $\int_b^a -\frac{3}{2}h(t)\,dt$.

$$\int_b^a -\frac{3}{2}h(t)dt = \frac{3}{2}\int_a^b h(t)dt$$

$$= \frac{3}{2}(2) = 3$$