The Fundamental Theorem of Calculus I

Suppose $f(x)$ is continuous on the interval $[a, b]$. If $F(x)$ is *any* antiderivative of $f(x)$ (so $F'(x) = f(x)$), then $\int_a^b f(x)\,dx = F(b) - F(a)$.

What do we mean by *any antiderivative*? We can choose any value for C that we want (we tend to choose zero, since it's the easiest to deal with).

Example 1: Compute the following integral.

$$\int_1^2 \left(2x + x^2\right)\,dx$$

Example 2: Compute the following integral.

$$\int_0^2 (3e^x + 2)\,dx$$

Example 3: Compute the following integral.

$$\int_0^{\frac{\pi}{4}} \left(2\sec^2(x) - 3\right)\, dx$$

Example 4: Find the area enclosed by the graphs of the following equations. $y = (2x^2 + 1)^2$, $y = 0$, $x = 0$, and $x = 8$.

DIY

1. Compute the following integral.

$$\int_1^9 \frac{x^2 + x^3}{\sqrt{x}}\, dx$$

2. Find the area enclosed by the graphs of the following equations. $y = x^2 + 4$, $y = 0$, $x = 1$, and $x = 2$.