The Fundamental Theorem of Calculus I

Suppose $f(x)$ is continuous on the interval $[a, b]$. If $F(x)$ is *any* antiderivative of $f(x)$ (so $F'(x) = f(x)$), then $\int_a^b f(x)\,dx = F(b) - F(a)$.

What do we mean by *any antiderivative*? We can choose any value for C that we want (we tend to choose zero, since it's the easiest to deal with).

For any constant C: $\int_a^b f(x)\,dx = (F(b)+c) - (F(a)+c)$

$= F(b) - F(a)$

$$\boxed{\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)}$$

Example 1: Compute the following integral.

$$\int_1^2 (2x + x^2)\,dx$$

$$= x^2 + \frac{1}{3}x^3 \Big|_1^2 = \left(4 + \frac{8}{3}\right) - \left(1 + \frac{1}{3}\right)$$

$$= \frac{20}{3} - \frac{4}{3}$$

$$= \frac{16}{3}$$

Example 2: Compute the following integral.

$$\int_0^2 (3e^x + 2)\,dx$$

$$= (3e^x + 2x)\Big|_0^2 = (3e^2 + 4) - (3 + 0)$$

$$= \boxed{3e^2 + 1}$$

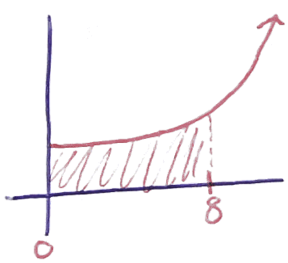
Example 3: Compute the following integral.

$$\int_0^{\frac{\pi}{4}} \left(2\sec^2(x) - 3\right)\, dx$$

$$= 2\tan(x) - 3x \Big|_0^{\pi/4}$$

$$= \left(2 - \frac{3\pi}{4}\right) - (0 - 0) = \boxed{2 - 3\frac{\pi}{4}}$$

Example 4: Find the area enclosed by the graphs of the following equations. $y = (2x^2+1)^2$, $y = 0$, $x = 0$, and $x = 8$.



$$A = \int_0^8 (2x^2+1)^2\, dx$$

$$= \int_0^8 (4x^4 + 4x^2 + 1)\, dx$$

$$= \frac{4}{5}x^5 + \frac{4}{3}x^3 + x \Big|_0^8$$

$$\approx \boxed{26905.06667}$$

DIY

1. Compute the following integral.

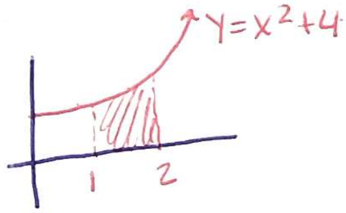
$$\int_1^9 \frac{x^2 + x^3}{\sqrt{x}}\, dx$$

$$= \int_1^9 x^{-1/2}(x^2 + x^3)\, dx = \int_1^9 (x^{3/2} + x^{5/2})\, dx$$

$$= \frac{2}{5} x^{5/2} + \frac{2}{7} x^{7/2} \Big|_1^9$$

$$= \frac{25272}{35} - \frac{24}{35} = \frac{25248}{35}$$

2. Find the area enclosed by the graphs of the following equations. $y = x^2 + 4$, $y = 0$, $x = 1$, and $x = 2$.



$$A = \int_1^2 (x^2 + 4)\, dx$$

$$= \frac{1}{3}x^3 + 4x \Big|_1^2$$

$$= \left(\frac{8}{3} + 8\right) - \left(\frac{1}{3} + 4\right)$$

$$= \frac{32}{3} - \frac{13}{3}$$

$$= \frac{19}{3}$$