The Fundamental Theorem of Calculus II

Example 1: The growth rate of the population of a city is $p'(t) = -200(5-t)$, where t is time in years. How does the population change from $t = 2$ to $t = 5$ years?

If I knew $p(t)$, I would find $p(5) - p(2)$.

Fundamental Theorem: $\int_a^b f(x)dx = F(b) - F(a)$, $F'(x) = f(x)$

$\Rightarrow P(5) - P(2) = \int_2^5 p'(t)dt$

$= \int_2^5 -200(5-t)\,dt = \int_2^5 (200t - 1000)dt$

$= (100t^2 - 1000t)\Big|_2^5$

$= (100(5)^2 - 1000(5)) - (100(2)^2 - 1000(2))$

$= -2500 - -1600$

$= -900$

$\boxed{\text{Population decreased by 900 people}}$

Example 2: A hose is turned on at 8:00 am and water starts to flow into a pool at a rate of $r(t) = 6\sqrt{t}$, where t is time in hours after 8:00 am. $r(t)$ is measured in ft^3/h.

(a) Find how much water flows into the pool between 9:00 am and 12:00 pm.

$V(4) - V(1) = \int_1^4 6\sqrt{t}\,dt = \int_1^4 6t^{1/2}dt$

$= \frac{2}{3} \cdot 6t^{3/2}\Big|_1^4 = 4t^{3/2}\Big|_1^4$

$= 4(4)^{3/2} - 4(1)^{3/2} = \boxed{28 \text{ ft}^3}$

(b) How many hours after 8:00 am will there be 100 cubic feet of water in the pool?

How long until $V = 100$ ft^3?

$100 = V(x) = V(x) - V(0) = \int_0^x 6\sqrt{t}\,dt = \int_0^x 6t^{1/2}dt$

$\Rightarrow 100 = \int_0^x 6t^{1/2}dt = 4t^{3/2}\Big|_0^x = 4x^{3/2} - 0$

$\Rightarrow 100 = 4x^{3/2} \Rightarrow 25 = x^{3/2} \Rightarrow x = 25^{2/3}$

$\Rightarrow \boxed{x \approx 8.5 \text{ hours}}$

Example 3: The velocity, in meters per minute, of a particle is $v(t) = 8t - 5$.

(a) Find the displacement of the particle from $t = 3$ to $t = 7$ minutes.

If $S(t)$ is position, then displacement =

$$S(7) - S(3) = \int_3^7 (8t - 5)\,dt$$

$$= (4t^2 - 5t)\Big|_3^7 = \left(4(7)^2 - 5(7)\right) - \left(4(3)^2 - 5(3)\right)$$

$$= 161 - 21 = 140\,m$$

(b) Find the time when the displacement is zero after the particle starts moving.

For what value x is $S(x) - S(0) = 0$?

$$0 = S(x) - S(0) = \int_0^x (8t - 5)\,dt$$

$$\Rightarrow 0 = 4t^2 - 5t \Big|_0^x = 4x^2 - 5x$$

$$0 = 4x^2 - 5x \Rightarrow 0 = (x)(4x - 5)$$

$x = 0$ $\quad x = 5/4$

5/4 minutes

DIY

1. The acceleration of a car, t seconds after the driver steps on the brake, is $a(t) = -(t-3)^2$. If distance is measured in meters, what is the decrease in velocity 3 seconds after the brake is applied?

$$V(3) - V(0) = \int_0^3 -(t-3)^2\,dt = \int_0^3 (-t^2 + 6t - 9)\,dt$$

$$= -\frac{1}{3}t^3 + 3t^2 - 9t \Big|_0^3$$

$$= \left(-\frac{1}{3}(3)^3 + 3(3)^2 - 9(3)\right) - 0$$

$$= -9$$

$\boxed{\text{Velocity has decreased by 9 m/s}}$