Exponential Growth

Example 1: Given that $\frac{dy}{dt} = 3y$ and $y(0) = 90$, find $y(t)$.

What function do we know whose derivative is 3 times itself? $y = Ce^{3t}$, C = constant

$y(0) = Ce^0 = 90 \Rightarrow C = 90$

$$\boxed{y = 90e^{3t}}$$

Exponential Growth Model

If $\frac{dy}{dt} = ky$, then $y = Ce^{kt}$, where C is the *initial amount* (when $t = 0$) and k is the *growth rate (proportionality constant).*

Example 2: The rate of change of the population of a town is $\frac{dP}{dt} = kP$, where P is the population after t years and k is the growth rate. If $P = 4000$ when $t = 3$ and $P = 5000$ when $t = 4$, what is the population when $t = 8$?

$P = Ce^{kt}$ Need to find C and K!

$4000 = Ce^{3k} \Rightarrow C = \frac{4000}{e^{3k}} \Rightarrow 5000 = \frac{4000}{e^{3k}} e^{4k}$

$5000 = Ce^{4k}$

$\Rightarrow \frac{5}{4} = e^k \Rightarrow k = \ln(5/4)$

$4000 = Ce^{3\ln(5/4)}$

$\Rightarrow 4000 = C(5/4)^3$

$\Rightarrow 2048 = C$

$P = Ce^{kt} \Rightarrow P = 2048e^{t\ln(5/4)}$

$P(8) = 2048e^{8\ln(5/4)}$

$\approx \boxed{12{,}207 \text{ people}}$

DIY

1. I deposited \$1000 in a savings account in which interest is compounded continuously. It takes 20 years for my deposit to double. This follows an exponential model where c = deposit amount and K = interest rate!

   (a) What is the annual interest rate?

$$P = ce^{Kt}$$

$$2000 = 1000e^{20K}$$

$$\Rightarrow 2 = e^{20K} \Rightarrow \ln(2) = 20K \Rightarrow K = \frac{\ln(2)}{20}$$

$$\approx .035$$

(3.5%)

   (b) How much money is in the account after 10 years?

$$P = Ce^{Kt}$$

$$P = 1000e^{\frac{1}{20}\ln(2)t}$$

$$P(10) = 1000e^{\frac{1}{20}\ln(2)\cdot 10}$$

$$\approx \boxed{\$1,414.21}$$