Exponential Decay

We use the same exponential model for exponential decay as we did for exponential growth.

$$\begin{aligned} \frac{dy}{dt} = ky \quad &\Rightarrow \quad y = Ce^{kt} \\ C \quad &= \quad \text{initial amount} \\ k \quad &= \quad \text{rate of decay} \end{aligned}$$

The only difference is that, in exponential growth, $k > 0$, and, in exponential decay, $k < 0$.

Example 1: The population P of a species of bird is decreasing at a rate that is proportional to the population itself. If $P = 5000$ when $t = 3$ and $P = 4000$ when $t = 4$, what is the population when $t = 9$?

Half-Life Formula

The *half-life* of a substance is the amount of time it takes for half of the initial amount to decay. The following relationship holds between the half-life and the rate of decay, k.

$$k = \frac{\ln\left(\frac{1}{2}\right)}{\text{half-life}}$$

Example 2: The radioactive isotope ^{239}Pu has a half-life of approximately 24,100 years. After 2,000 years there are 5 grams of ^{239}Pu left.

(a) What was the initial quantity?

(b) How much remains after 5,000 years?

DIY

1. The radioactive isotope ^{14}C has a half-life of approximately 5,715 years. A piece of charcoal contains only 25% as much of the radioactive carbon as a piece of modern charcoal. How old is this sample of charcoal?