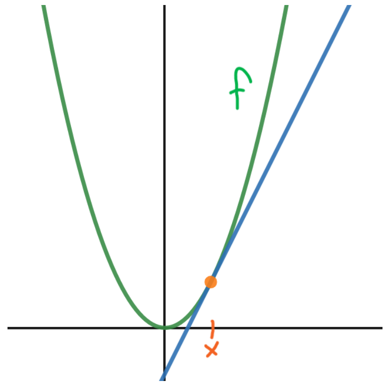
The Derivative

Tangent Lines

The *tangent line* to a function $f(x)$ at a point c is a line that touches the graph of $f(x)$ at the point $(c, f(c))$. Note, this means that the point $(c, f(c))$ is always on the tangent line.

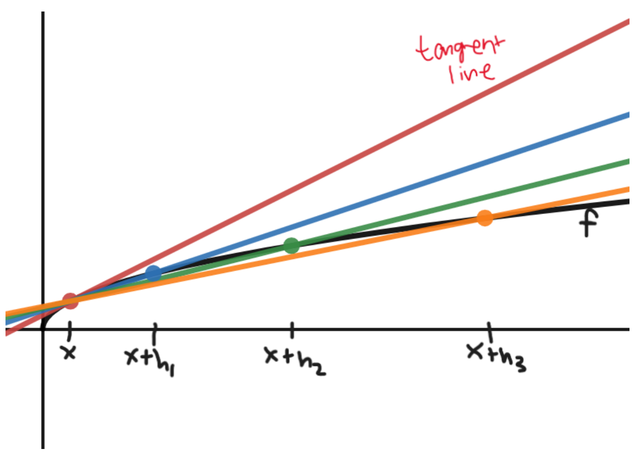


We can think of this as taking the slope of the graph at the point $(x, f(x))$ and extending it into a line with the same slope.

Slope of the Tangent Line to $f(x)$ at $x =$ Slope of the graph of $f(x)$ at the point $(x, f(x))$.

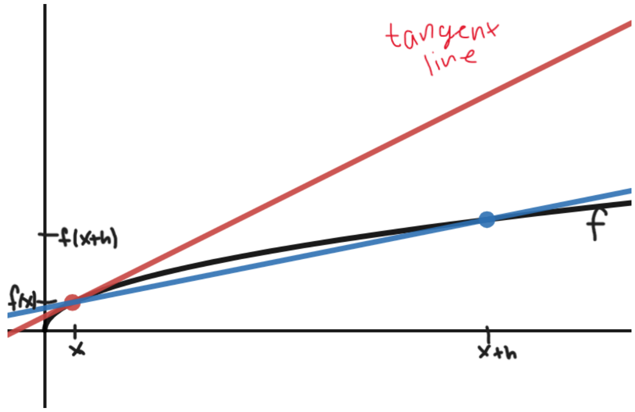
Tangent Lines as Limits of Secant Lines

A *secant line* is a line that passes through the graph of $f(x)$ in two points.



What happens to points $x + h_1$, $x + h_2$, and $x + h_3$ as the secant lines approach the tangent line?

Finding the Slope of the Tangent Line Using Slopes of Secant Lines



What is the slope of the secant line in the picture above?

Take the limit as $h \to 0$ to get the slope of the tangent line in the picture above.

The Derivative

We define the *derivative* of a function $f(x)$ at the point x to be the slope of the tangent line at any point x.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Other Notation: If $y = f(x)$, then $y' = f'(x) = \frac{dy}{dx} = \frac{d}{dx} f(x)$.

Geometric Interpretation: The slope of the tangent line to $f(x)$ at the point $x = c$ is equal to the derivative of f at the point c ($f'(c) =$ slope of the tangent line at c).

Example 1: Given $f(x) = 3x^2 + 1$, use the limit of the difference quotient to find $f'(x)$.

Example 2: Find the equation of the tangent line to the graph of $f(x) = 3x^2 + 1$ at the point $x = 2$.

Example 3: The derivative of a function $g(x)$ is found by computing

$$g'(x) = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h}.$$

What could $g(x)$ be?

DIY

1. Find the slope of the tangent line to the graph of $f(x) = \frac{2}{3x}$ at the point $x = c$.