Some Differentiation Rules

Basic Rules

- Derivative of a Constant: $\frac{d}{dx}[c] = 0$.

  Example 1: $\frac{d}{dx}[12] = 0$ $\frac{d}{dx}[1,000,000] = 0$

- The Power Rule: (powers of x) $\frac{d}{dx}[x^n] = nx^{n-1}$.

  Example 2: $\frac{d}{dx}[x^5] = 5x^4$

  $\frac{d}{dx}[\sqrt[3]{x}] = \frac{d}{dx}[x^{1/3}] = 1/3\, x^{-2/3}$

- Constant Multiple: Let c be a constant. $\frac{d}{dx}[cf(x)] = c\left[\frac{d}{dx}(f(x))\right]$.

  Example 3: $\frac{d}{dx}[4x^3] = 4\frac{d}{dx}[x^3] = 4(3x^2) = 12x^2$

- Sum/Difference Rule: $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

  Example 4: $\frac{d}{dx}\left[3\sqrt[3]{x} - \frac{2}{x^3} + 7x\right] = \frac{d}{dx}[3x^{1/3}] - \frac{d}{dx}[2x^{-3}] + \frac{d}{dx}[7x]$

  $= 3\left[\frac{d}{dx}(x^{1/3})\right] - 2\frac{d}{dx}(x^{-3}) + 7\frac{d}{dx}(x)$

  $= 3(1/3)x^{-2/3} + 6x^{-4} + 7x^0$

  $= \boxed{\frac{1}{x^{2/3}} + \frac{6}{x^4} + 7}$

Sine and Cosine

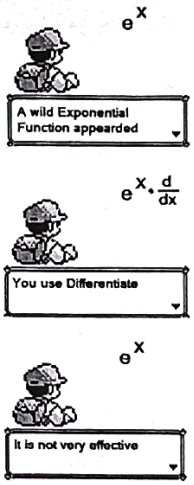
$\frac{d}{dx}\sin(x) = \cos(x)$
$\frac{d}{dx}\cos(x) = -\sin(x)$

Example 5: $\frac{d}{dx}[2\sin(x) - 3\cos(x)] = 2\frac{d}{dx}\sin(x) - 3\frac{d}{dx}\cos(x)$

$= \boxed{2\cos(x) + 3\sin(x)}$

Exponential Function

$\frac{d}{dx}e^x = e^x$



Example 6: $\frac{d}{dx}[7e^x] =$ $7\frac{d}{dx}e^x = 7e^x$

DIY

1. Find $f'(x)$ for the following function.

$$f(x) = \frac{3x^5 + x^{1.5}}{\sqrt{x}}$$

$f(x) = \frac{3x^5}{\sqrt{x}} + \frac{x^{1.5}}{\sqrt{x}} = \frac{3x^5}{x^{1/2}} + \frac{x^{1.5}}{x^{1/2}} = 3x^{9/2} + x$

$\Rightarrow \boxed{f'(x) = \frac{27}{2}x^{7/2} + 1}$

2. Find the equation of the tangent line to the graph of $y = 4\cos(x)$ at $x = \frac{\pi}{2}$.

Point: $(\pi/2, 0)$

Slope: $y' = -4\sin(x) \rightarrow y'(\pi/2) = -4\sin(\pi/2) = -4$

$y - 0 = -4(x - \pi/2)$

$\boxed{y = 2\pi - 4x}$

3. If $g(x) = (x-1)(x+2)$, find all values of x so that $g'(x) = -1$.

$g(x) = (x-1)(x+2) = x^2 + x - 2$

$\Rightarrow g'(x) = 2x + 1$

Want to find values of x so that

$g'(x) = 2x + 1 = -1$ (solve for x)

$\Rightarrow 2x = -2$

$\Rightarrow \boxed{x = -1}$