Instantaneous Rates of Change

Recall that the slope of the secant line to $f(x)$ at the points x and $x+h$ is

$$\frac{f(x+h)-f(x)}{h}.$$

This is the *average rate of change* of the function f over the interval $[x, x+h]$.

Taking the limit as $h \to 0$ gives the *(instantaneous) rate of change* at the point x.

$$\begin{aligned}\text{(Instantaneous) rate of change} &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= f'(x)\\ &= \text{the derivative of } f \text{ at } x\end{aligned}$$

Example 1: The population of a culture of bacteria is given by $P(t) = 7t^2 + 4t + 1500$.

(a) Find the equation for the rate of change of the population after t hours.

$P'(t) = 14t + 4$

(b) What is the rate of change after 4 hours?

$P'(4) = 14(4) + 4 = 60$

Velocity

The rate of change of position is *velocity.* If $s(t)$ is a function giving the position of an object at time t, then the velocity of that object at time t is $v(t) = s'(t)$.

Example 2: The height of a ball t seconds after being thrown into the air is given by $s(t) = -16t^2 + 51t$.

(a) Find the velocity function ($v(t)$).

$s'(t) = v(t) = -32t + 51$

(b) What is the velocity of the ball when $t = 2$?

$v(2) = -32(2) + 51 = -13$

DIY

1. If a rock is thrown upward on Mars, its height (in meters) after t seconds is given by $s(t) = 16t - 1.86t^2$. At what time is the velocity of the rock equal to -2.6 m/s?

$s'(t) = v(t) = 16 - 3.72t$

$16 - 3.72t = -2.6$

$\Rightarrow 18.6 = 3.72t$

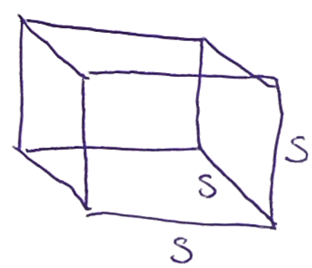
$\Rightarrow \boxed{5 \text{ seconds} = t}$

2. Find the rate of change of the volume of a cube with respect to the length s of a side. What is the rate of change of the volume when $s = 4$?

$V = s^3$

$\Rightarrow V'(s) = 3s^2$

$V'(4) = 3(4)^2 = \boxed{48}$



3. The population of a pride of lions over time (in years) is given by $P(t) = 150\,(1 + 0.5t + 0.08t^2)$. What is the growth rate (in lions per year) when $t = 5$ years?

$P'(t) = 150(0.5 + 0.16t)$

$P'(5) = 150(0.5 + 0.16(5)) = \boxed{195 \text{ lions per year}}$