The Product Rule

Example 1: Find $f'(x)$ given that $f(x) = 2x^2\left(3x^4 + 5\right)$.

$f(x) = 6x^6 + 10x^2$

$f'(x) = 36x^5 + 20x$

What if we take the derivatives of each factor and multiply those together?

$f'(x) \stackrel{?}{=} 4x(12x^3 + 0) = 48x^4$ ... not even close!

$$\cancel{\frac{d}{dx}[f(x)g(x)] \neq \left[\frac{d}{dx}f(x)\right]\left[\frac{d}{dx}g(x)\right]}$$ Bad news!

We can't simply multiply the derivatives of each factor together. Don't do this!!

The Product Rule

$$\boxed{\frac{d}{dx}[f(x)g(x)] = \left[\frac{d}{dx}f(x)\right]g(x) + f(x)\left[\frac{d}{dx}g(x)\right]}$$

Example 1 (revisited): Use the product rule to find $f'(x)$ given that $f(x) = 2x^2\left(3x^4 + 5\right)$.

$f'(x) = 4x(3x^4 + 5) + 2x^2(12x^3)$

$= 12x^5 + 20x + 24x^5 = 36x^5 + 20x.$ ✓

Example 2: If $y = 3e^x \sin(x)$, find $y'\left(\frac{\pi}{2}\right)$.

$$y' = 3e^x \sin(x) + 3e^x \cos(x)$$

$$y'\left(\tfrac{\pi}{2}\right) = 3e^{\pi/2}\sin(\pi/2) + 3e^{\pi/2}\cos\left(\tfrac{\pi}{2}\right)$$

$$= 3e^{\pi/2}\cdot 1 + 3e^{\pi/2}\cdot 0$$

$$= \boxed{3e^{\pi/2}}$$

Keep answers exact unless you are asked for a decimal answer!

DIY

1. Find the x-values at which $y = 2x^3e^x$ has a horizontal tangent line.

slope $= 0 \Rightarrow y' = 0$

$$y' = 6x^2e^x + 2x^3e^x$$
$$= e^x(6x^2 + 2x^3)$$
$$= e^x\, 2x^2(3+x)$$

$y' = e^x\, 2x^2(3+x) = 0$

never zero $\quad x=0 \quad x=-3$

$\boxed{x = 0 \text{ and } x = -3}$

2. Find the equation of the tangent line to $y = 4x\cos(x)$ at $x = \pi$.

Point: $(\pi, y(\pi)) = (\pi, 4\pi\cos(\pi)) = (\pi, -4\pi)$

Slope: $y' = 4\cos x + 4x(-\sin x)$

$$= 4\cos x - 4x\sin x$$

$$y'(\pi) = 4\cos(\pi) - 4(\pi)\sin(\pi) = -4$$

$$y + 4\pi = -4(x-\pi) \Rightarrow y = -4x + 4\pi - 4\pi \Rightarrow \boxed{y = -4x}$$