The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

If we need to take the derivative of two functions being divided, we cannot simply divide the derivative of the numerator by the derivative of the denominator;

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] \neq \frac{f'(x)}{g'(x)}.$$

Example 1: Compute the derivative of the following function.

$$y = \frac{\sin(x) + x}{2x + 1}$$

Example 2: Compute the derivative of the following function.

$$y = \frac{ae^x}{(a^2 + \sqrt{x})}, \quad \text{where } a \text{ is a constant}$$

Derivatives of Other Trigonometric Functions

- $\frac{d}{dx}\tan(x) = \sec^2(x)$
- $\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$
- $\frac{d}{dx}\cot(x) = -\csc^2(x)$
- $\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$

Example 3: Find the derivative of the function $y = 5\cos(x)\cot(x)$.

DIY

1. Find the derivative of the following function at $t = 7$.

$$y = \frac{2 + e^t}{3 - e^t}$$

2. Find the equation of the tangent line to the graph of $y = 3e^x \sec(x)$ at $x = 0$.

3. Find the derivative of the following function.

$$y = \frac{\cos(x)}{\cot(x) + \sec(x)}$$