

Simplify your final answers. Show all relevant work for each problem. Little to no work, even with a correct answer, will receive little to no credit.

1. How long does it take for a sum of money to double if it is compounded continuously with an interest rate of 8%? Round your answer to the nearest tenth.

Compounded continuously $\Rightarrow y = Ce^{kt}$ where
 $C =$ initial amount, $k =$ interest rate $= 0.08$

double
initial
amount \rightarrow

$$2C = Ce^{0.08t}$$

$$2 = e^{0.08t}$$

$$\ln(2) = 0.08t$$

$$\frac{\ln(2)}{0.08} = t$$

$$\Rightarrow \boxed{t \approx 8.7 \text{ years}}$$

2. What is the half-life of a radioactive substance if it takes 5 years for one-third of the substance to decay? Round your answer to the nearest tenth.

Radioactive decay $\Rightarrow y = Ce^{kt}$ where $C =$ initial amount
and k is rate of decay.

$\frac{1}{3}$ has decayed
 $\Rightarrow \frac{2}{3}$ left.

$$\frac{2}{3}C = Ce^{5k}$$

$$\frac{2}{3} = e^{5k}$$

$$\ln\left(\frac{2}{3}\right) = 5k$$

$$\underline{\frac{1}{5} \ln\left(\frac{2}{3}\right) = k}$$

$$\text{So, } y = Ce^{\frac{1}{5} \ln\left(\frac{2}{3}\right)t}$$

half-life = amount of time for $\frac{1}{2}$ original amount
to decay (so there's $\frac{1}{2}$ left).

$$\frac{1}{2}C = Ce^{\frac{1}{5} \ln\left(\frac{2}{3}\right)t}$$

$$\frac{1}{2} = e^{\frac{1}{5} \ln\left(\frac{2}{3}\right)t}$$

$$\ln\left(\frac{1}{2}\right) = \frac{1}{5} \ln\left(\frac{2}{3}\right)t$$

$$\frac{5 \ln\left(\frac{1}{2}\right)}{\ln\left(\frac{2}{3}\right)} = t$$

$$\Rightarrow \boxed{t \approx 8.5 \text{ years}}$$