MA 16010 Quiz 2

Show all relevant work for each problem. Little to no work, even with a correct answer, will receive little to no credit.

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1. Determine whether $f(x)$ is continuous or discontinuous at $x = 0$ and $x = 1$. If it is discontinuous at either of these points, state the type of discontinuity (i.e., is it a hole, a jump, or a vertical asymptote).

$$f(x) = \begin{cases} x^2 + 3, & x \leq 0 \\ 7x + 3, & 0 < x < 1 \\ -4x + 3, & x \geq 1 \end{cases}$$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (x^2+3) = 3$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (7x+3) = 3$

$\lim_{x \to 0} f(x) = 3$

$f(0) = 3$

f is continuous at $x=0$.

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (7x+3) = 10$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (-4x+3) = -1$

f has a jump discontinuity at $x=1$.

2. Find the derivative of $g(x) = x^2 + 2$ by using the limit process. You may not use any other method.

$g(x+h) = (x+h)^2 + 2 = x^2 + 2xh + h^2 + 2$

$g(x+h) - g(x) = x^2 + 2xh + h^2 + 2 - (x^2+2) = 2xh + h^2$

$$\frac{g(x+h) - g(x)}{h} = \frac{2xh + h^2}{h} = \frac{h(2x+h)}{h} = 2x + h$$

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \to 0} (2x+h) = 2x$$

$$\boxed{g'(x) = 2x}$$