MA 165

Calculus : Early Trancendentals 8th Edition

by James Stewart

Chapter 1 Functions and Models

- lesson 1 : 1.1
- lesson 2 : 1.3 + App. D
- lesson 3 : 1.4 + 1.5

Lesson 1 §1.1 Four Ways to Represent a Function

indep. var.

Function f is a rule that assigns to each element $x \in D$ domain

dep. var.

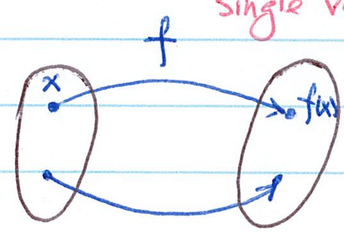
eactly one element $f(x) \in E$. range

single value function

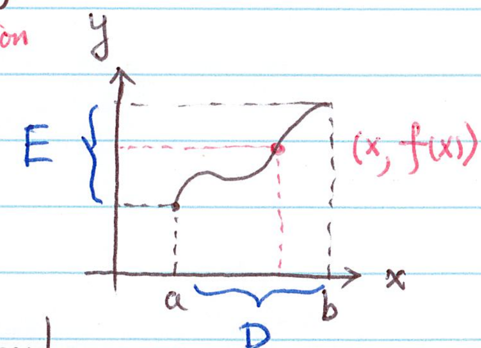
input output

$x \longrightarrow \boxed{f} \longrightarrow f(x)$

machine diagram



arrow diagram



graph

$\{(x, f(x)) \mid x \in D\}$

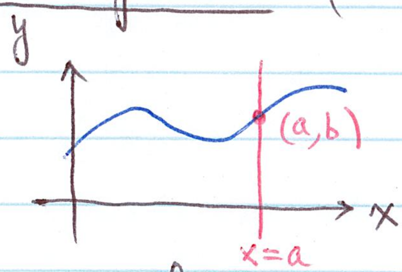
- Examples for evaluation, domain, and range
  (P21) #29, 31, 38

Representation of Functions

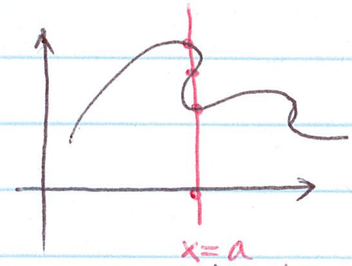
- verbally (in words)
- numerically (table)
- visually (graph)
- algebraidly (explicit formula)

examples (P22) #57, #59 (word problems)

- single-value function (vertical line test)



a function



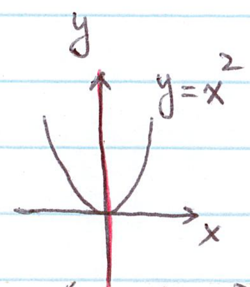
not a function

- Piecewise Defined Functions (evaluation and sketch)

$$f(x) = \begin{cases} 1-x & \text{if } x \le -1 \\ x^2 & \text{if } x > -1 \end{cases}, \qquad f(x) = |x|$$

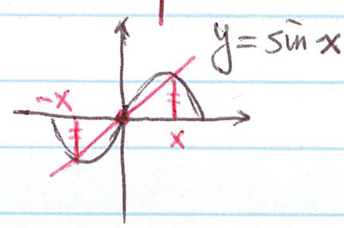
- Symmetry

even function $f(-x) = f(x)$



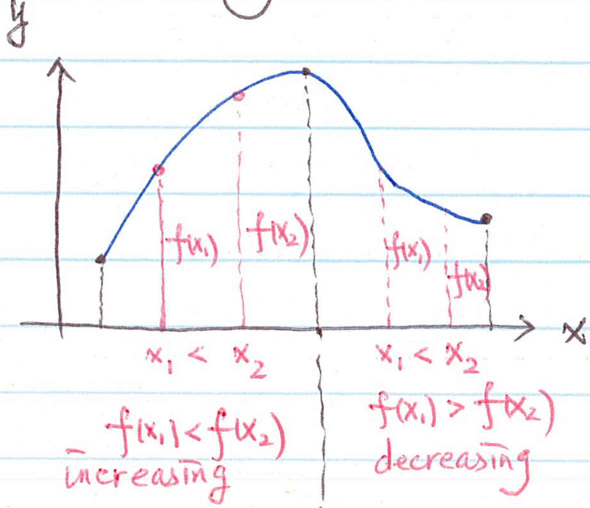
sym w.r.t. y-axis

odd function $f(-x) = -f(x)$



sym w.r.t origin

- Increasing and Decreasing Functions



Lesson 2 (§1.3 and App. D)

§1.3 New Functions from Old Functions (use ebook for lecture)

- Transformations of Functions (shifting, stretching, reflecting)

shifting the graph of f : see Fig. 1 shift $y = f(x) \pm c$ or $f(x \pm c)$

stretching and reflecting : see Fig. 2 stretch $y = c f(x)$ or $f(cx)$

see Fig. 3 reflect $y = -f(x)$ or $f(-x)$

examples 1, 2, 3

- Combinations of Functions

$(f \underset{\times}{\pm} g)(x) = f(x) \underset{\times}{\pm} g(x)$ $\quad \mathrm{dom}(f \underset{\times}{\pm} g) = \mathrm{dom}(f) \cap \mathrm{dom}(g)$

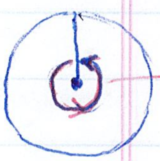
$(f/g)(x) = f(x)/g(x)$ $\quad \mathrm{dom}(f/g) = \{x \in \mathrm{dom}(f) \cap \mathrm{dom}(g) \mid g(x) \neq 0\}$

$(f \circ g)(x) = f(g(x))$ $\quad \mathrm{dom}(f \circ g) = \{x \in \mathrm{dom}(g) \mid g(x) \in \mathrm{dom}(f)\}$

examples #35, 41

Appendix D Trigonometry

• Angles

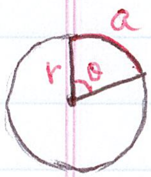


degree or radian

$\wr$

in calculus

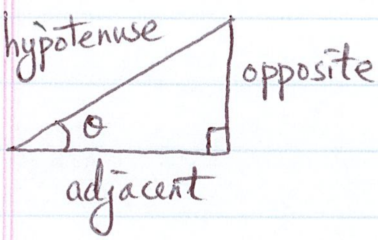
$\pi \text{ rad} = 180° \Longrightarrow \begin{cases} 1 \text{ rad} = \left(\frac{180}{\pi}\right)° \\ 1° = \frac{\pi}{180} \text{ rad} \end{cases}$



$$\frac{\theta}{2\pi} = \frac{a}{2\pi r} \Longrightarrow \begin{cases} \theta = \frac{a}{r} \\ a = r\theta \end{cases}$$

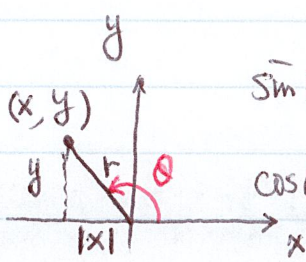
• Trigonometric Functions

θ — an acute angle



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \Longrightarrow \begin{cases} \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\text{opp}}{\text{adj}} \\ \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\text{adj}}{\text{opp}} \\ \sec\theta = \frac{1}{\cos\theta} = \frac{\text{hyp}}{\text{adj}} \\ \csc\theta = \frac{1}{\sin\theta} = \frac{\text{hyp}}{\text{opp}} \end{cases}$$

θ — general angle (obtuse or neg.)



$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \Longrightarrow \begin{cases} \sin(\pi - \theta) = \sin\theta \\ \cos(\pi - \theta) = -\cos\theta \end{cases}$$

• Trigonometric Identities

$$\begin{cases} \sin^2\theta + \cos^2\theta = 1 \\ \sin(x+y) = \sin x \cos y + \cos x \sin y \\ \cos(x+y) = \cos x \cos y - \sin x \sin y \end{cases}$$

#42 Prove $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

#67 Find all x s.t. $2\sin^2 x = 1$

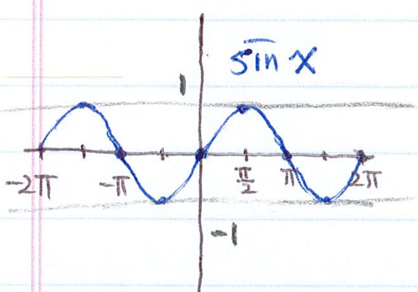
periodic function

$$\sin(\theta + 2\pi) = \sin\theta, \quad \cos(\theta + 2\pi) = \cos\theta$$

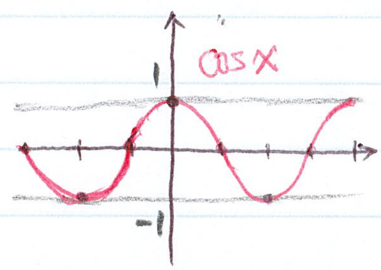
symmetry

odd $\sin(-\theta) = -\sin\theta$, even $\cos(-\theta) = \cos\theta$

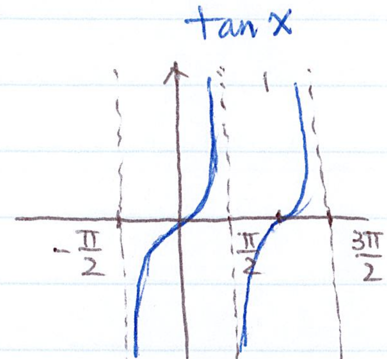
- **Graphs of Trigonometric Functions**



odd, period 2π



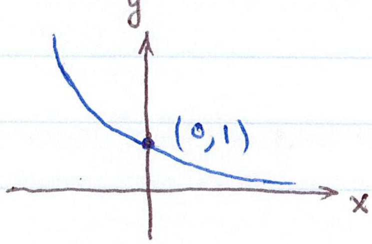
even, period 2π

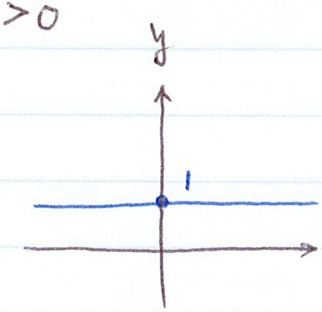


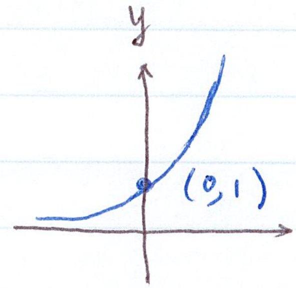
odd, period π

§1.4 Exponential Functions

$f(x) = b^x$ with $b > 0$ $\qquad$ $\text{dom}(f) = (-\infty, +\infty)$







$0 < b < 1$ $\qquad$ $b = 1$ $\qquad$ $b > 1$

$\text{range}(f) = (0, +\infty)$ $\qquad$ $\text{range}(f) = \{1\}$ $\qquad$ $\text{range}(f) = (0, +\infty)$

Law of Exponents

$$b^{x+y} = b^x \cdot b^y, \quad b^{x-y} = \frac{b^x}{b^y}, \quad (b^x)^y = b^{xy}, \quad (ab)^x = a^x \cdot b^x.$$

The Number e

$e \approx 2.71828$

$$\frac{d}{dx}(2^x)\bigg|_{x=0} \approx 0.7, \quad \frac{d}{dx}(3^x)\bigg|_{x=0} \approx 1.1, \quad \boxed{\frac{d}{dx} e^x \bigg|_{x=0} = 1}$$

Example 4 Sketch $y = \frac{1}{2}e^{-x} - 1$, dom = ?, range = ?

#18 (P53) Starting with the graph of $y = e^x$, find the equation of the graph that results from

(a) reflecting about the line $y = 4$ $\qquad$ $y = -e^x +$

(b) " " " " $x = 2$

§1.5 Inverse Functions and Logarithms

$y = f(x)$ single valued

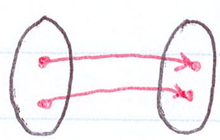
$x = f^{-1}(y)$ what guarantees f^{-1} being single valued?

Def (1-to-1) $f(x)$ is a 1-to-1 function

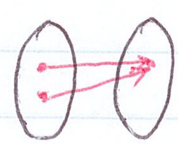
$\iff \forall x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$

$\iff f(x_1) = f(x_2) \implies x_1 = x_2$

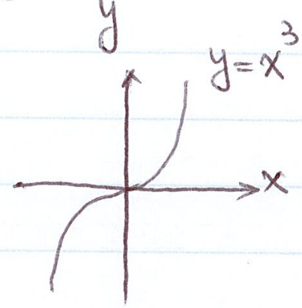
$\iff$ horizontal line test



1-to-1



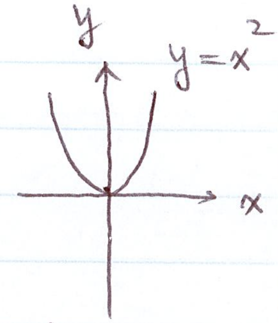
not 1-to-1



1-to-1

$0 = x_1^3 - x_2^3 = (x_1 - x_2)(x_1^2 + x_1 x_2 + x_2^2)$

$\implies x_1 = x_2$



not 1-to-1

$y(-1) = y(1) = 1$

Def (inverse function) f is 1-to-1 function with $\mathrm{dom}(f) = A$ and $\mathrm{range}(f) = B$

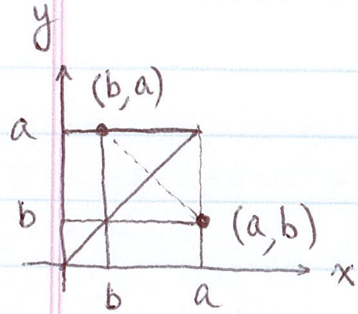
$\implies$ the inverse function f^{-1} is defined by

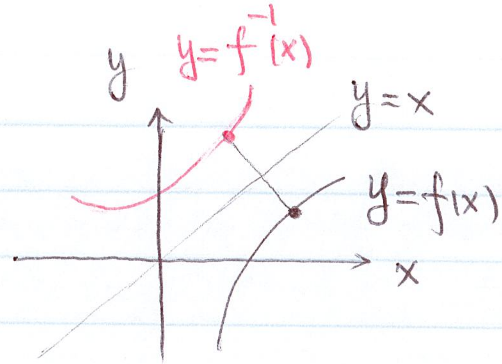
$$f^{-1}(y) = x \iff y = f(x)$$

with $\mathrm{dom}(f^{-1}) = B$ and $\mathrm{range}(f^{-1}) = A$.

$$\boxed{\begin{aligned} f^{-1}(f(x)) &= x \\ f(f^{-1}(y)) &= y \end{aligned}}$$

Ex. 4 Find the inverse function of $f(x) = x^3 + 2$ and $f(x) = x^2 - x$ for $x \geq \frac{1}{2}$





f^{-1} is obtained by reflecting the graph of f w.r.t. the line $y=x$.

Logarithmic Functions (inverse of exponential functions)

$$y=b^x, \quad b>0 \text{ and } b\neq 1 \iff x=f^{-1}(y)=\log_b y, \quad y>0$$

1-to-1

$$\Downarrow$$

$$b^y=x \iff y=\log_b x, \text{ for } x>0$$

logarithmic function with base b.

$$\boxed{\begin{array}{ll} \log_b b^x = x & \forall x\in R \\ b^{\log_b x} = x & \forall x>0 \end{array}}$$

Law of Logarithms $x, y>0$

$z=\log_b(xy)$, $z_1=\log_b x$, $z_2=\log_b y$

$\Rightarrow b^z=xy$, $b^{z_1}=x$, $b^{z_2}=y$

$\Rightarrow b^z=b^{z_1}\cdot b^{z_2}=b^{z_1+z_2} \Rightarrow z=z_1+z_2$

$$\log_b(xy)=\log_b x+\log_b y, \quad \log_b\left(\frac{x}{y}\right)=\log_b x-\log_b y$$

$$\log_b x^r = r\log_b x$$

$y=\log_b x \Rightarrow b^y=x$

$\Rightarrow y\ln b=\ln x$

$\Rightarrow y=\ln x/\ln b$

Natural Logarithms

$$\ln x=\log_e x, \quad \boxed{\log_b x=\frac{\ln x}{\ln b}}, \quad \ln e=1$$