

# Chapter 1 Functions and Models

## §1.1 Four Ways to Represent a Function

indep. var.

Def  $f$  is a rule that assigns to each element

$x \in D$

exactly one element  $f(x) \in E$ .

range

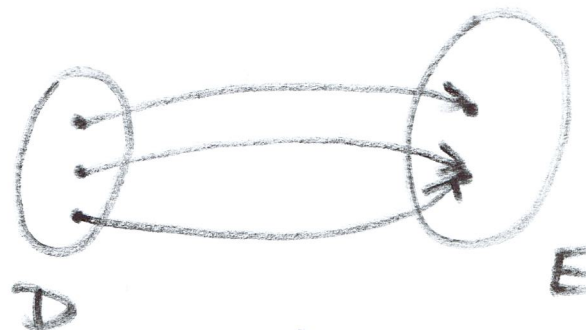
indep. var.

domain

single value function  
(vertical line test)

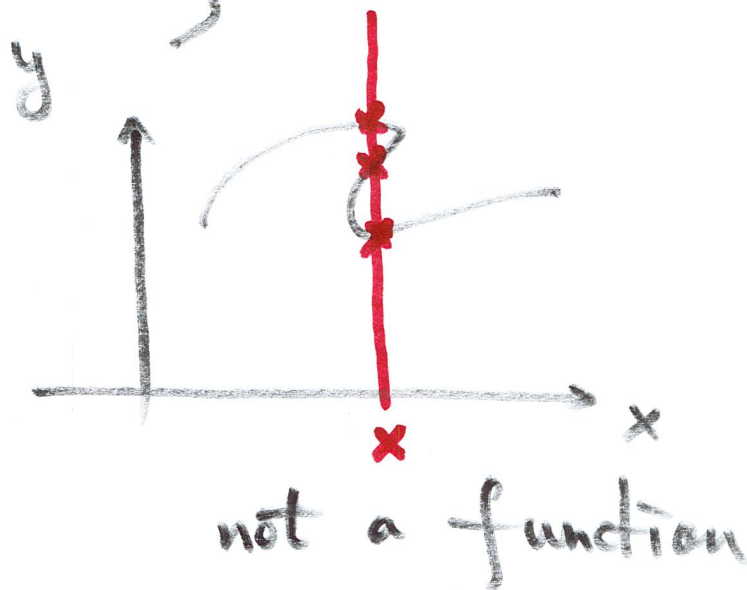
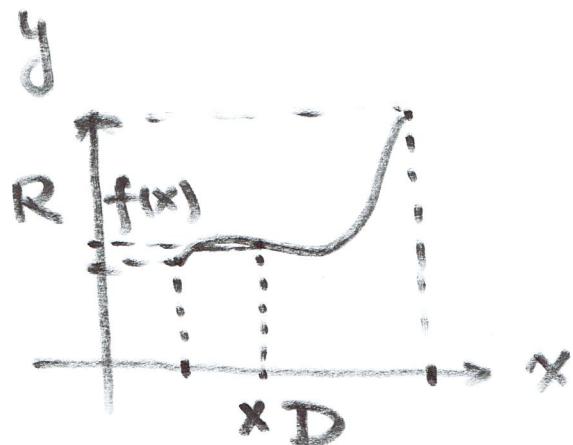


machine diagram



arrow diagram

graph  $\{(x, f(x)) \mid x \in D\}$



Ex. 1  $f(x) = 2x + x^4$

$$f(1) = 2 \cdot 1 + 1^4 = 3, \quad f(-1) = 2 \cdot (-1) + (-1)^4 = -2 + 1 = -1$$

$$f(a) = 2a + a^4$$

$$\begin{aligned} \frac{f(a+h) - f(a)}{(a+h) - a} &= \frac{[2(a+h) + (a+h)^4] - [2a + a^4]}{h} \\ &= \{2h + (a+h)^4 - a^4\} / h \end{aligned}$$

Ex. 2  $f(x) = \frac{1}{\sqrt{x^2-1}}$ , find its domain

(1) denominator  $\sqrt{x^2-1} \neq 0$

$$\Rightarrow x^2 - 1 \neq 0 \Rightarrow x^2 \neq 1 \Rightarrow |x| \neq 1 \Rightarrow x \neq \pm 1$$

(2) inside of square root  $x^2 - 1 \geq 0$

$$\Rightarrow x^2 \geq 1 \Rightarrow |x| \geq 1 \Rightarrow x \leq -1 \text{ or } x \geq 1$$

$$\left\{ \begin{array}{l} x \neq \pm 1 \\ x \leq -1 \text{ or } x \geq 1 \end{array} \right.$$

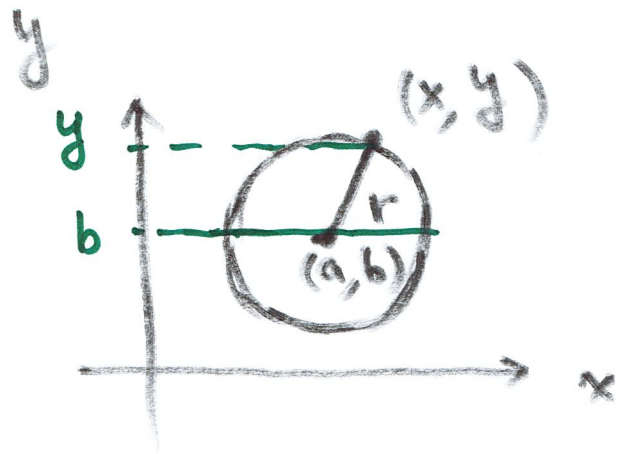
$$\Rightarrow x < -1 \text{ or } x > 1$$

$$\Rightarrow D = (-\infty, -1) \cup (1, +\infty)$$

$$= \{x \mid x < -1 \text{ or } x > 1\}$$

Ex. 3 Find an expression for the function whose graph is the given curve: the top half of the circle

$$(x-a)^2 + (y-b)^2 = r^2$$



• solve the eq. for  $y$

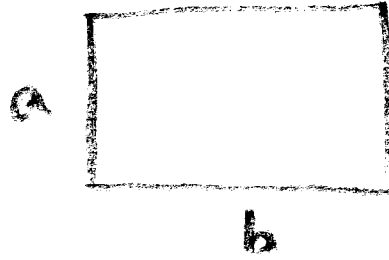
$$(y-b)^2 = r^2 - (x-a)^2$$

$$\Rightarrow |y-b| = \sqrt{r^2 - (x-a)^2}$$

$$\Rightarrow y-b = \pm \sqrt{r^2 - (x-a)^2}$$

top circle  $\Rightarrow y = b + \sqrt{r^2 - (x-a)^2}$

Ex. 4 (#57) rectangle



Given perimeter 20 m

Find area as a function of one side

$$A = ab$$

$$20 = 2a + 2b \Rightarrow a = \frac{20 - 2b}{2} = 10 - b$$

$$\Rightarrow A = ab = (10 - b)b$$