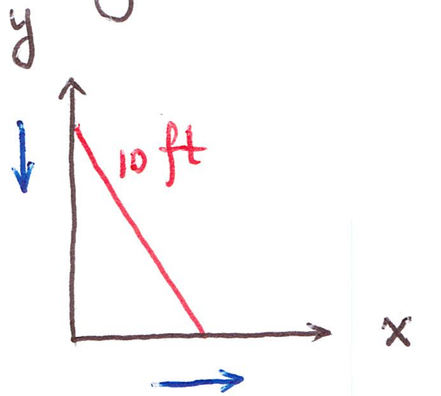
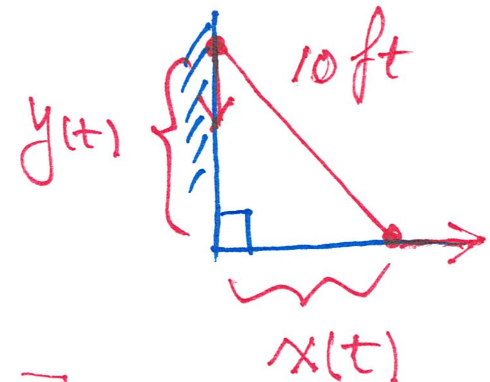
Ex. 2 A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?



Given $\dfrac{dx}{dt} = 1 \text{ ft/s}$

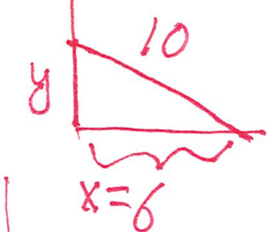
Unknown $\left.\dfrac{dy}{dt}\right|_{x=6} = ?$



relation

$$\frac{d}{dt}\left[x^2(t) + y^2(t) = 10^2\right] \quad \text{relation}$$

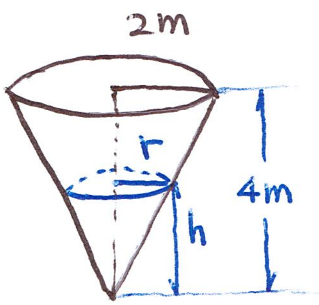
$$2x(t)\cdot\frac{dx}{dt} + 2y(t)\frac{dy}{dt} = 0$$



$$\left.\frac{dy}{dt}\right|_{x=6} = \frac{-2x(t)\frac{dx}{dt}}{2y(t)} = -\left.\frac{x(t)}{y(t)}\frac{dx}{dt}\right|_{x=6}$$

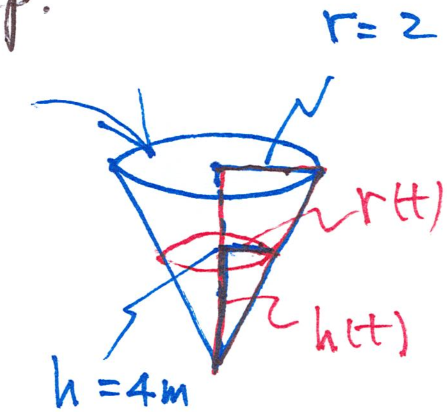
$$= -\frac{6}{\sqrt{10^2-6^2}}\cdot 1 = -\frac{6}{\sqrt{16\cdot 4}} = -\frac{6}{8} = -\frac{3}{4}$$

Ex. 3 A water tank has the shape of an inverted circular cone with base radius 2 m and height 4 m. If water is being pumped into the tank at a rate of 2 m^3/min, find the rate at which the water level is rising when the water is 3 m deep.



Given $\frac{dV(t)}{dt} = 2 \ m^3/min$

Unknown $\left.\frac{dh(t)}{dt}\right|_{h=3} = ?$



relation

$V = \frac{1}{3}\pi r^2 h$

$$V(t) = \frac{1}{3}\pi r(t)^2 h(t)$$

$$= \frac{1}{3}\pi \left(\frac{1}{2}h\right)^2 h$$

$$= \frac{1}{12}\pi h^3(t)$$

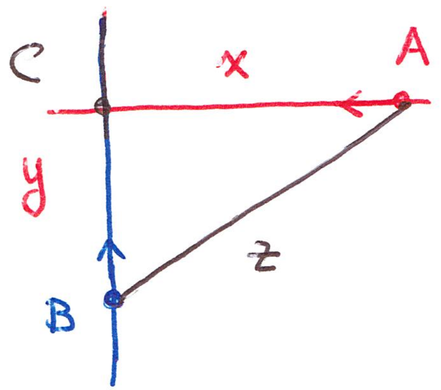
$$\frac{r}{2} = \frac{h}{4}$$

$$r = \frac{1}{2}h$$

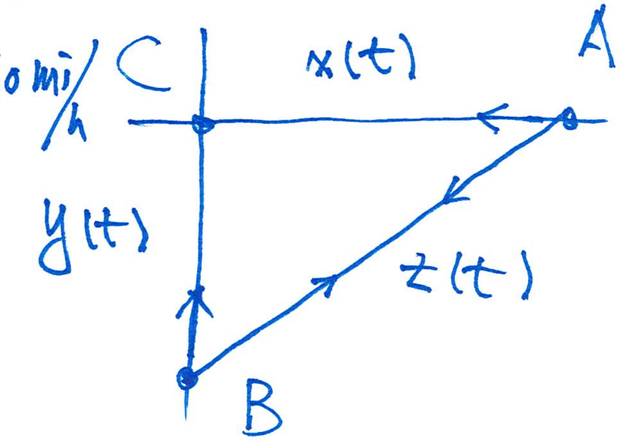
$$\frac{dV}{dt} = \frac{1}{12}\pi \, 3h^2(t) \cdot \frac{dh}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$$

$$\left.\frac{dh}{dt}\right|_{h=3} = \left.\frac{4}{\pi h^2}\frac{dV}{dt}\right|_{h=3} = \frac{4}{\pi \cdot 9} \cdot 2 = \frac{8}{9\pi} \ m/min$$

Ex. 4 Car A is traveling west at 50 mi/h and car B is traveling north at 60 mi/h. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection?



Given $\frac{dx}{dt} = -50\ \text{mi/h}, \quad \frac{dy}{dt} = -60\ \text{mi/h}$



Unknown $\left.\frac{dz}{dt}\right|_{(0.3=x,\ y=0.4)} = ?$

relation

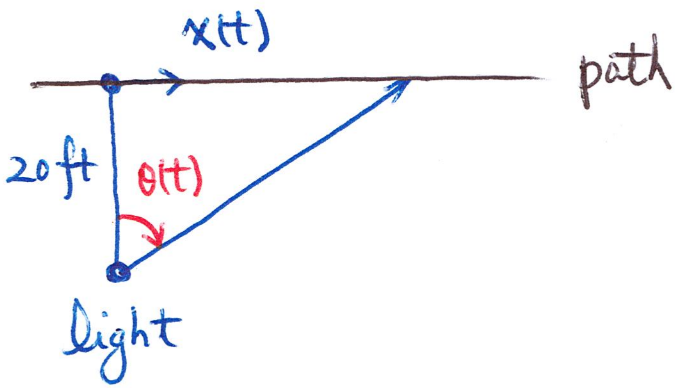
$$\frac{d}{dt}\left[z^2(t) = x^2(t) + y^2(t)\right]$$ relation

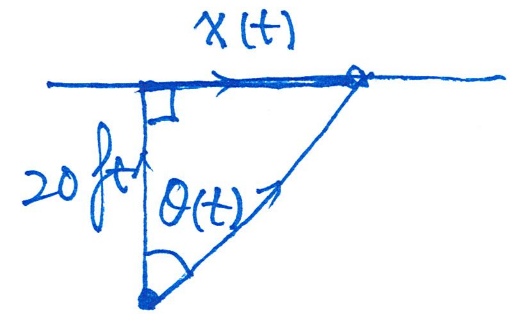
$$2z \cdot \frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\left.\frac{dz}{dt}\right|_{(x=0.3,\ y=0.4} = \frac{1}{z}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)\Bigg|_{(x=0.3,\ y=0.4)}$$

$$= \frac{1}{\sqrt{(0.3)^2+(0.4)^2}}\left[0.3\cdot(-50) + 0.4\,(-60)\right]$$

Example 5 A man walks along a straight path at a speed of 4 ft/s. A searchlight is located on the ground 20 ft from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 ft from the point on the path closest to the searchlight?





Given $\dfrac{dx(t)}{dt} = 4$ ft/s

Unknown $\left.\dfrac{d\theta(t)}{dt}\right|_{x=15} = ?$

relation

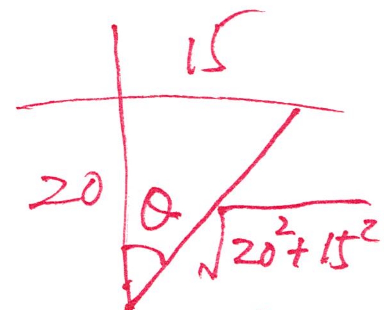
relation $\dfrac{d}{dt}\left[\tan\theta(t) = \dfrac{x(t)}{20}\right]$

$\sec\theta = \dfrac{1}{\cos\theta}$

$$\sec^2\theta \, \frac{d\theta}{dt} = \frac{1}{20}\,\frac{dx}{dt}$$

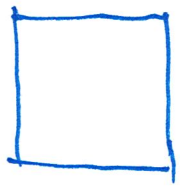
$$\left.\frac{d\theta(t)}{dt}\right|_{x=15} = \left.\frac{\cos^2\theta}{20}\cdot\frac{dx}{dt}\right|_{x=15} = \frac{1}{20}\cdot\left(\frac{4}{5}\right)^2\cdot 4$$

$$= \frac{16}{125} \text{ ft/s}$$



$$\cos\theta = \frac{20}{\sqrt{400+225}} = \frac{20}{25} = \frac{4}{5}$$

#3



$a(t)$

$\frac{da}{dt} = 6\ cm/s$ Given

$\left.\frac{dA}{dt}\right|_{A=16} = ?$ unknown

relation $A(t) = a^2(t)$ ⟶ $\left.\frac{dA}{dt}\right|_{A=16} = 2a(t)\left.\frac{da}{dt}\right|_{A=16}$

$16 = a^2 \Rightarrow \underline{a = 4}$

$= 2 \cdot 4 \cdot 6 = 48\ cm^2/s$

#4

$w(t)$

$l(t)$

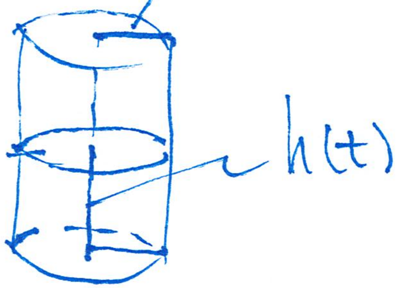
$A(t) = w(t)\,l(t)$, $\frac{dw}{dt} = 3\ cm/s$

$\left.\frac{dA}{dt}\right|_{(l=20,\ w=10)} = ?$ $\frac{dl(t)}{dt} = 8\ cm/s$

$= \left.\frac{dw}{dt}\cdot l(t) + w(t)\frac{dl(t)}{dt}\right|_{(l=20,\ w=10)} = 3\cdot 20 + 10\cdot 8$

$= 140\ cm^2/s$

#5



<u>given</u> $\frac{dV}{dt} = 3\ m^3/s$

<u>unknown</u> $\frac{dh}{dt}\Big|$

$$V(t) = \pi r^2 \cdot h(t) = \pi \cdot 25 \cdot h(t)$$

$$\frac{dV}{dt} = 25\pi \frac{dh}{dt} \boxed{\cancel{= 25 \cdot 3 \cdot \pi}}$$

$$\frac{dh}{dt} = \frac{1}{25\pi}\frac{dV}{dt} = \frac{3}{25\pi}$$