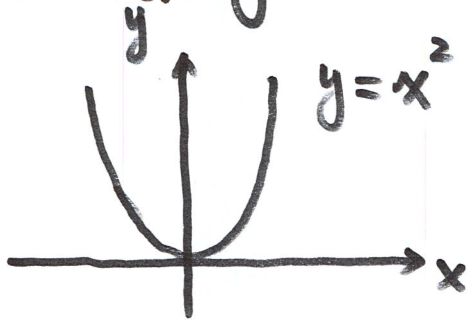
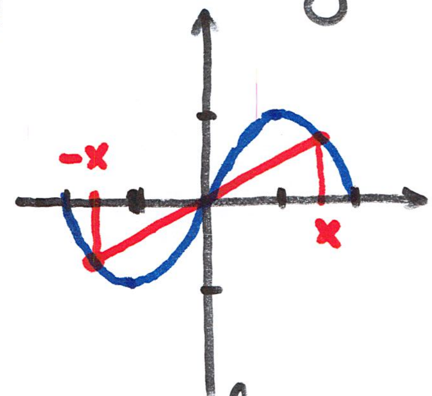
Symmetry

sym w.r.t. y-axis



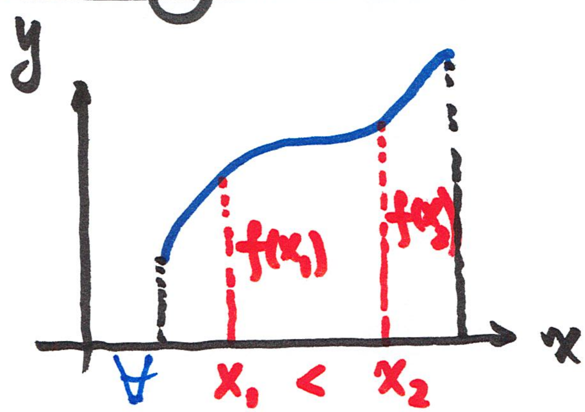
$f(-x)=f(x)$ even function

sym w.r.t. origin



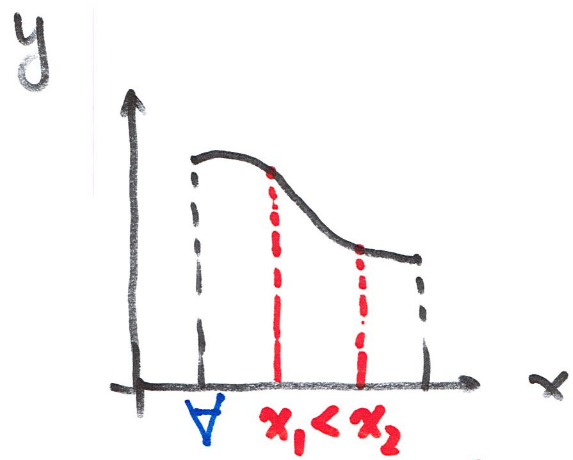
$f(-x)=-f(x)$ odd function

Increasing and Decreasing Functions



$f(x_1) < f(x_2)$

increasing



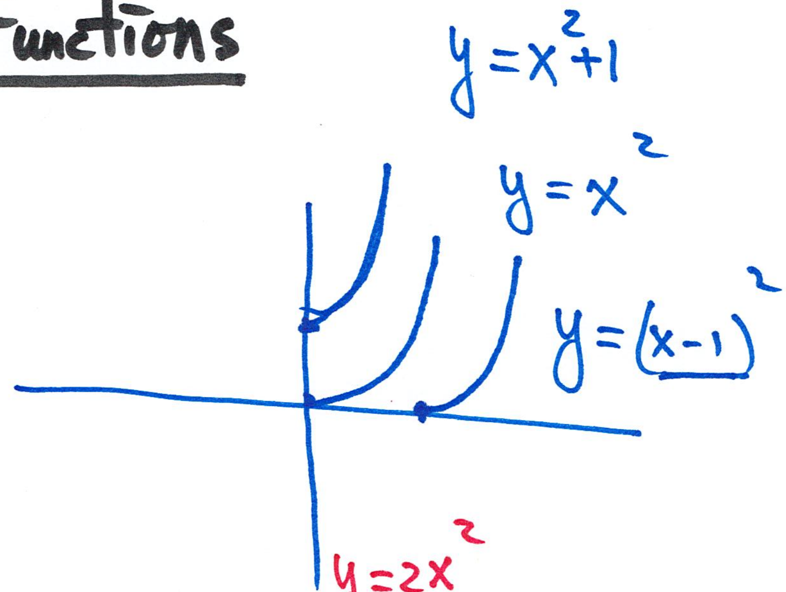
$f(x_1) > f(x_2)$

decreasing

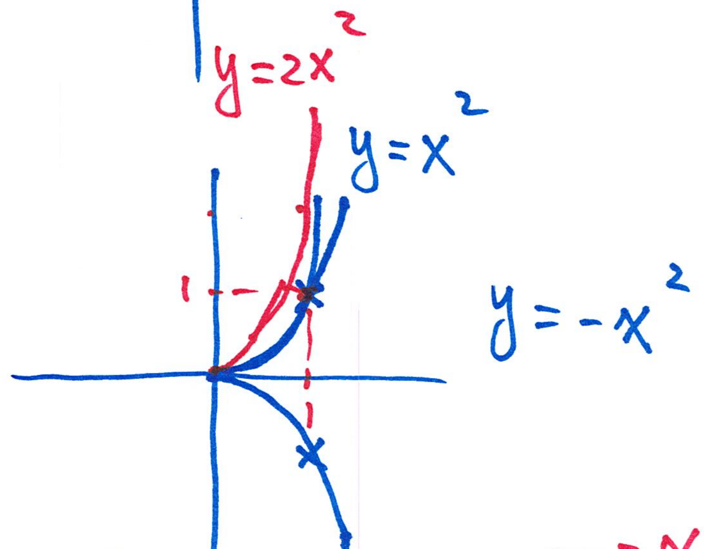
§1.3 New Functions from Old Functions

• Transformations

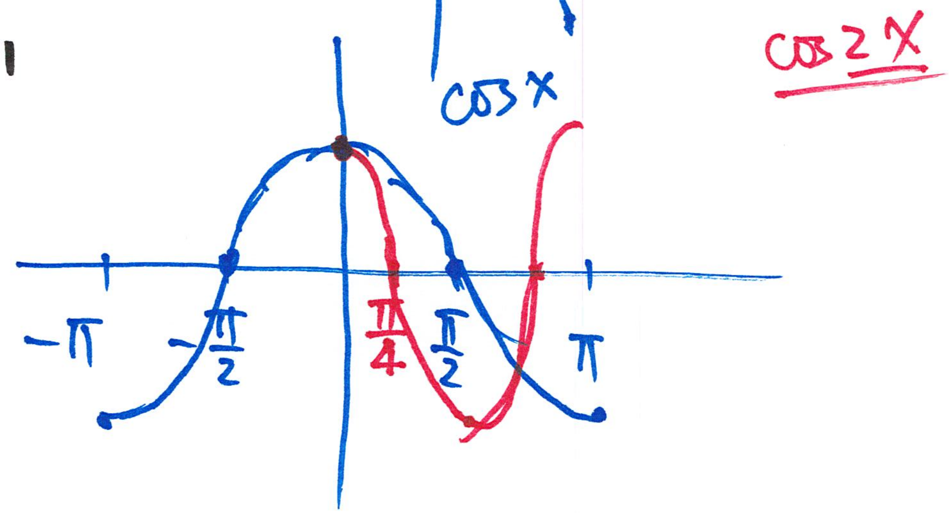
shift $\begin{cases} y = f(x) \pm c \\ y = f(x \pm c) \end{cases}$



stretch $\begin{cases} y = c\,f(x) \ \begin{cases} c>1 \\ c<1 \end{cases} \\ y = f(cx) \ \begin{cases} c>1 \\ c<1 \end{cases} \end{cases}$

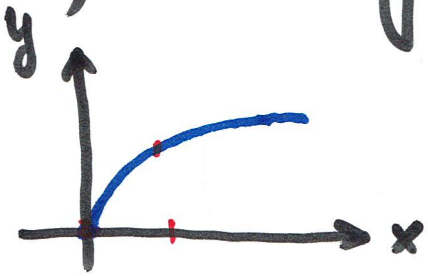


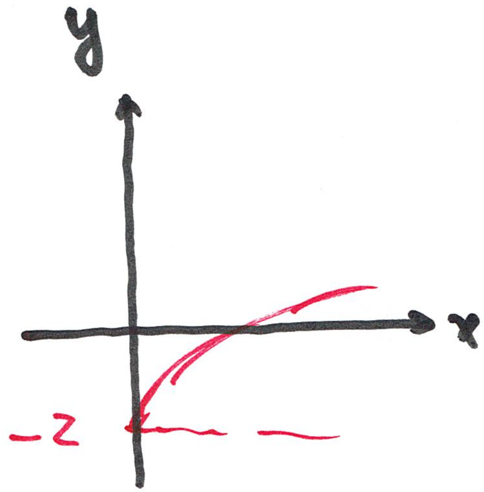
reflect $\begin{cases} y = -f(x) \\ y = f(-x) \end{cases}$



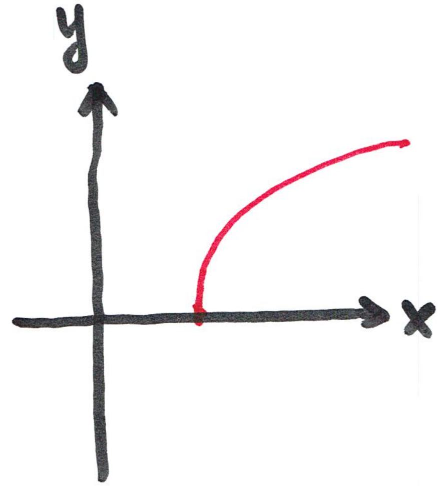
Ex. 1 Given graph of $y=\sqrt{x}$, use transf. to graph

$y=\sqrt{x}$

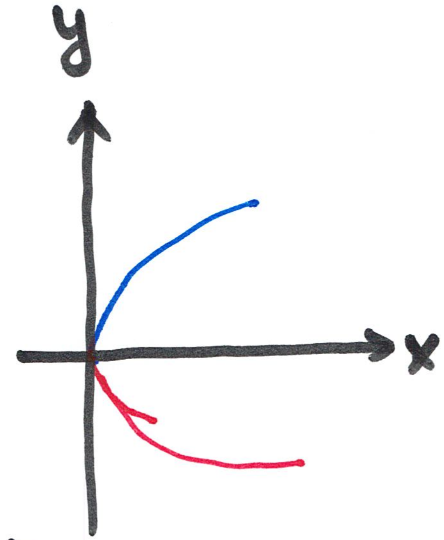




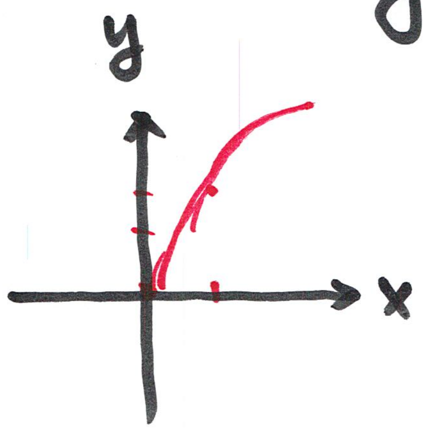
$y=\sqrt{x}-2$



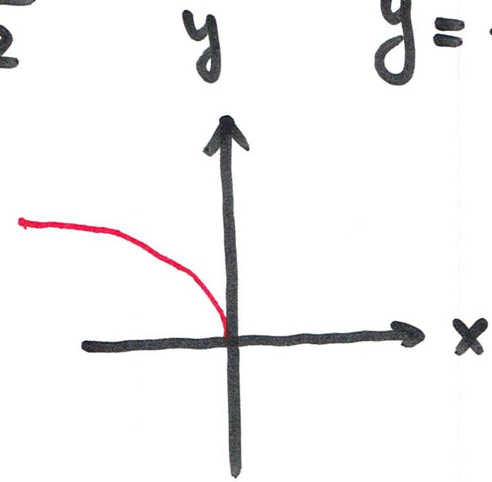
$y=\sqrt{x-2}$



$y=-\sqrt{x}$



$y=2\sqrt{x}$



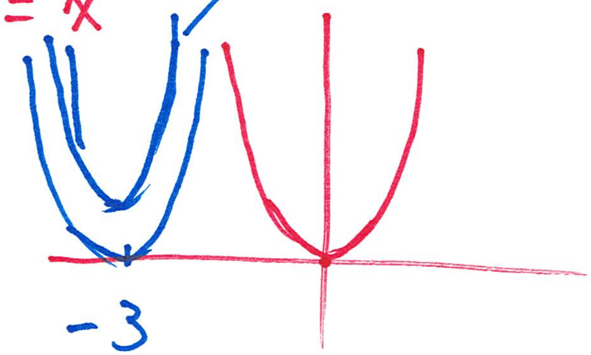
$y=\sqrt{-x}$

Ex. 2 Sketch the graph of $y = f(x) = x^2 + 6x + 10$

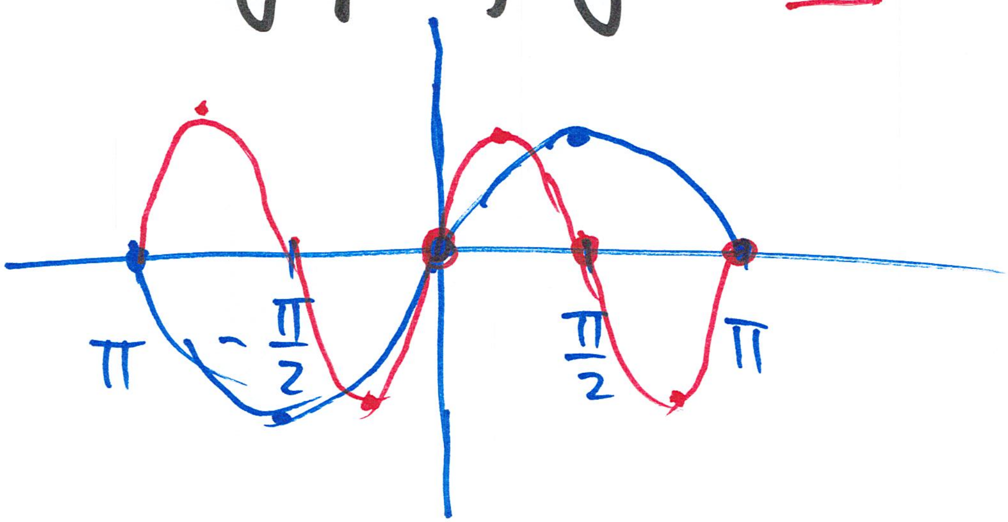
$y = (x^2 + 6x + 9) + 1 = (\underline{x+3})^2 \underline{+1}$

$y = (x+3)^2 + 1$ $\quad$ $y = (\underline{x+3})^2$

$y = x^2$



Ex. 3 Sketch the graph of $y = \sin \underline{2x}$



Combinations

$$\left(f \underset{\times}{\overset{+}{-}} g\right)(x) = f(x) \underset{\times}{\overset{+}{-}} g(x) \qquad \mathrm{dom}\left(f \underset{\times}{\overset{+}{-}} g\right) = \mathrm{dom}(f) \cap \mathrm{dom}(g)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \qquad \mathrm{dom}\left(\frac{f}{g}\right) = \left\{ x \in \mathrm{dom}(f) \cap \mathrm{dom}(g) \;\middle|\; g(x) \neq 0 \right\}$$

$$(f \circ g)(x) = f\left(\underline{g(x)}\right) \qquad \mathrm{dom}(f \circ g) = \left\{ x \in \cancel{g(x)}\ \mathrm{dom}(g) \;\middle|\; g(x) \in \mathrm{dom}(f) \right\}$$

Ex. (#35) $f(x) = \sqrt{x+1}$, $g(x) = 4\underline{x} - 3$, find

$$f \circ g = \sqrt{(4x-3)+1} \qquad f \circ f = \sqrt{\sqrt{x+1}+1}$$

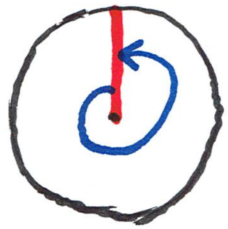
$$g \circ f = 4\sqrt{x+1} - 3 \qquad g \circ g = 4(4x-3) - 3$$

Appendix D Trigonometry

- Angles

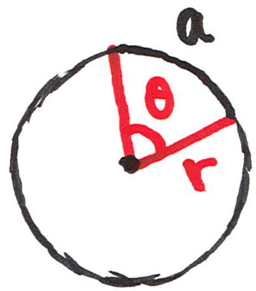
degree

or radian



$$360^\circ = 2\pi \Rightarrow \pi \text{ rad} = 180^\circ$$

$$1 \text{ rad} = \left(\frac{180}{\pi}\right)^\circ, \qquad 1^\circ = \frac{\pi}{180}$$



arc length a

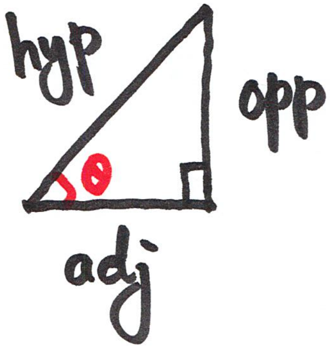
central angle θ

$$\frac{a}{2\pi r} = \frac{\theta}{\cancel{360}\,2\pi} \Rightarrow \frac{a}{r} = \theta$$

$$a = r\theta$$

Trigonometric Functions

θ — an acute angle



$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

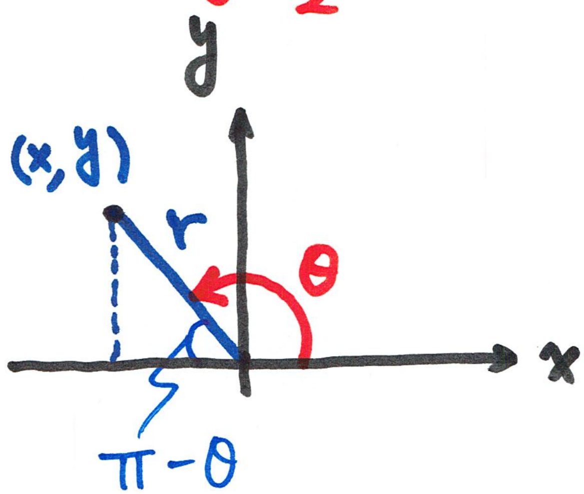
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

θ — general angle

$\theta > \frac{\pi}{2}$ or $\theta < 0$



$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\Rightarrow$$

$$\sin(\pi - \theta) = \sin\theta$$

$$\cos(\pi - \theta) = -\cos\theta$$

Trigonometric Identities

$\sin^2\theta + \cos^2\theta = 1$,

$\sin(x+y) = \sin x \cos y + \sin y \cos x$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

#42 Prove $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$\text{LHS} = \cos\left(\frac{\pi}{2} + (-x)\right) = \cos\frac{\pi}{2}\cos(-x) - \sin\left(\frac{\pi}{2}\right)\sin(-x)$

$= -1 \cdot \sin(-x) = \sin x$

#67 Find all x s.t. $2\sin^2 x = 1$

$y = \sin x$

$2y^2 = 1$

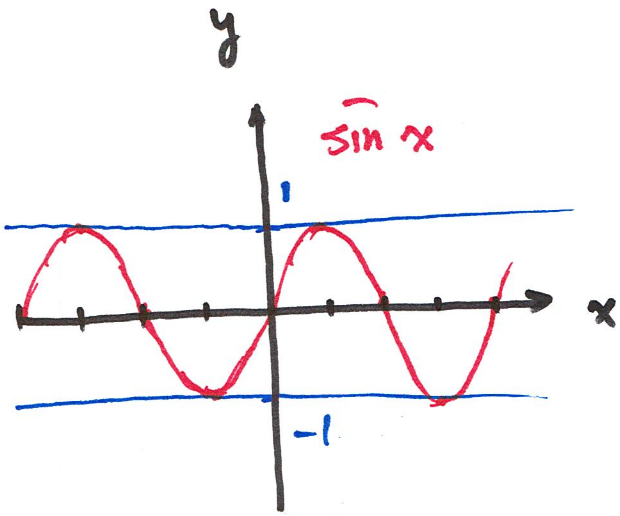
$0 = \sin^2 x - \frac{1}{2} = \left(\sin x + \sqrt{\frac{1}{2}}\right)\left(\sin x - \sqrt{\frac{1}{2}}\right)$

$a^2 - b^2 = (a+b)(a-b)$

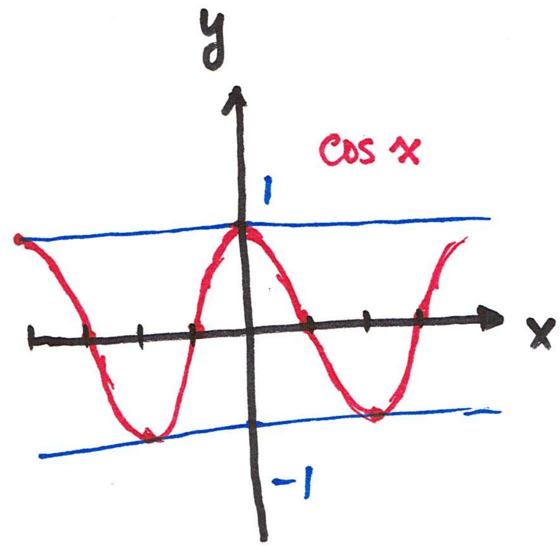
$\Rightarrow$ $\sin x = \frac{\sqrt{2}}{2}$

$\sin x = -\frac{\sqrt{2}}{2}$

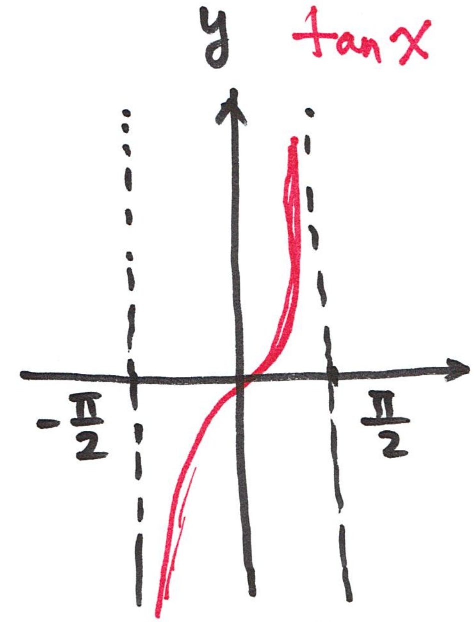
Graph of Trigonometric Functions



odd, period 2π



even, period 2π



odd period π