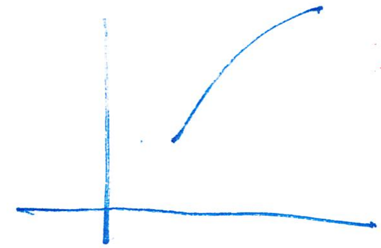
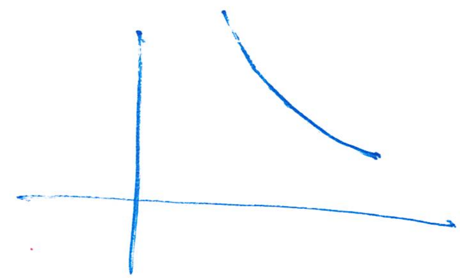
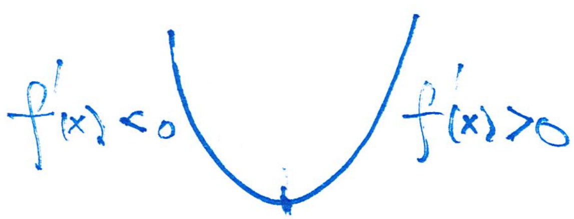
$f'(x) > 0 \Longrightarrow f(x) \nearrow$



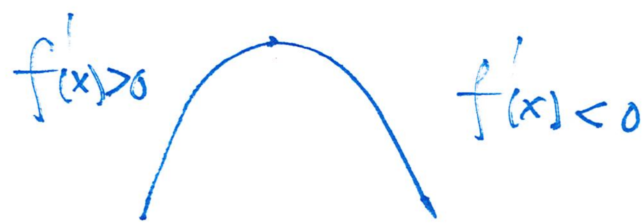
$f'(x) < 0 \Longrightarrow f(x) \searrow$



1st der. test



local min



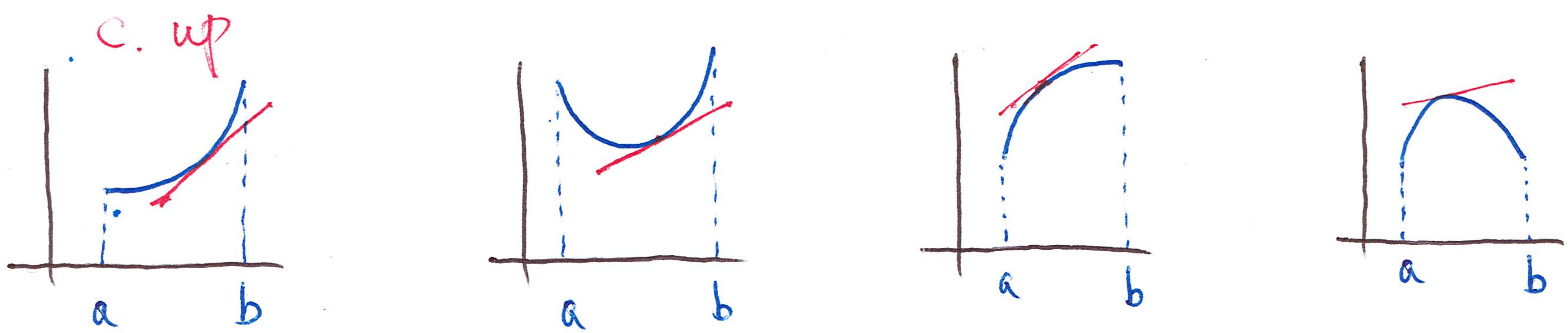
local max

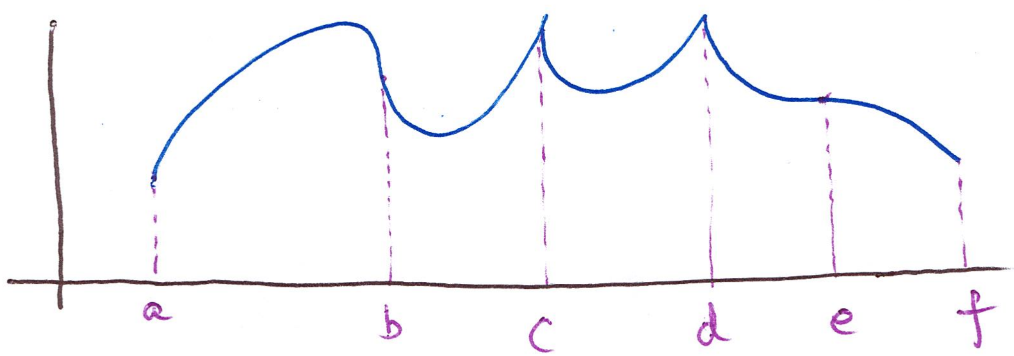
• what does $f''(x)$ say about f ?

Def. (concave upward/downward)

(1) f is concave upward on an interval $I \iff$ the graph of f lies above all of its tangen

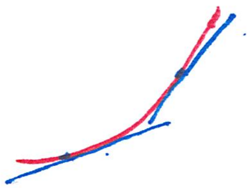
(2) f is concave downward on an interval $I \iff$ the graph of f lies below all of its tangents.

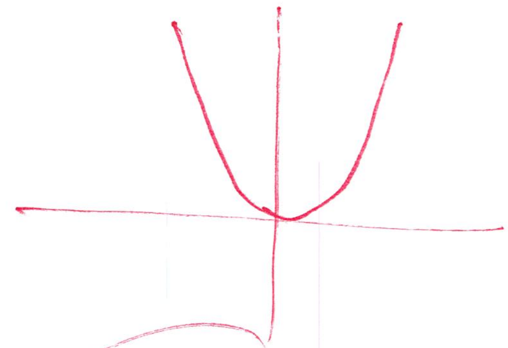




$f''(x) > 0 \implies \left(f'(x)\right)' > 0 \implies f'(x) \nearrow \implies$ concave up

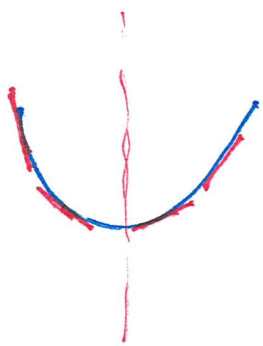
$f(x) = x^2$

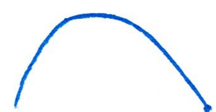




$f'(x) \nearrow$

$f'' = 2 > 0$





Concavity Test

(a) $f''(x) > 0 \ \forall x \in I \implies$ the graph of f is concave up on I

(b) $f''(x) < 0 \ \forall x \in I \implies$ the graph of f is concave down on I

Def. (inflection point)

A point P on a curve $y = f(x)$ is an inflection point $\iff$ (1) f is continuous at P

(2) the curve changes its concavity.

Ex. 5 Sketch a possible graph of a function f that satisfies the following conditions:

(i) $f'(x) > 0$ on $(-\infty, 1)$

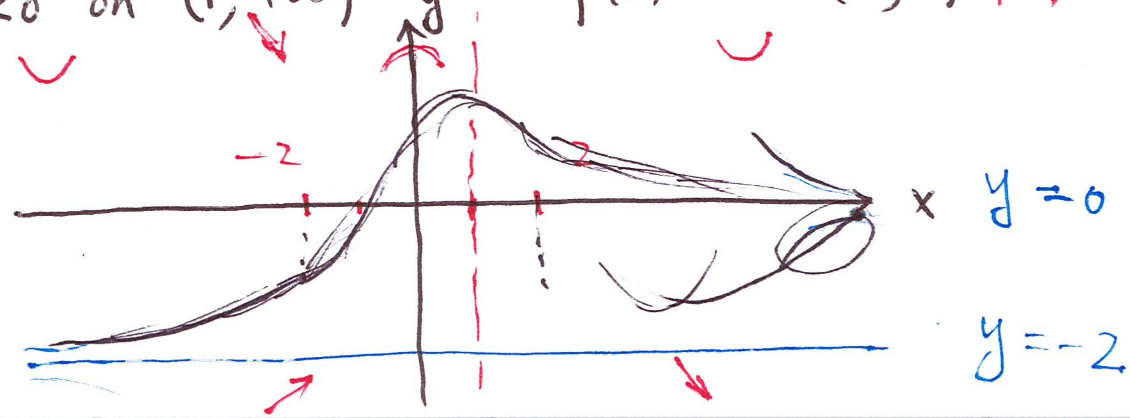
$f'(x) < 0$ on $(1, +\infty)$

(ii) $f''(x) > 0$ on $(-\infty, -2) \cup (2, \infty)$

$f''(x) < 0$ on $(-2, 2)$

(iii) $\lim\limits_{x \to -\infty} f(x) = -2 = y$

$\lim\limits_{x \to \infty} f(x) = 0 = y$



The Second Derivative Test

f'' is continuous near c.

(a) $f'(c)=0$ and $f''(c)>0 \implies f$ has a local minimum at c.

(b) $f'(c)=0$ and $f''(c)<0 \implies f$ has a local maximum at c.

Ex. 6 Discuss the curve $y=x^4-4x^3$ with respect to concavity, points of inflection, and local maxima and minima. Use this information to sketch the curve.

critical number $0=f'(x)=4x^3-12x^2=4x^2(x-3) \implies c=0,\ 3$

$f'(x)$	$-$ ↘	0	$-$ ↘	0	$+$ ↗
		0		3	
				l. min	

$f''(x)=12x^2-24x=12x(x-2)$

$f''(0)=0$

$f''(x)$	$+$		$-$		$+$
	∪	0	∩	2	∪

4

$f''(3)=12\cdot3\cdot1>0 \implies f(3)=3^3-4\cdot3^3$
$=3^3(3-4)$
$=-27$
l. min