# Exam 2

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|---|---|---|
| Average | 62 | |
| # of students | 923 | |
| A | 100 - 84 | 24.2% |
| B | 76 - 68 | 23.5% |
| C | 60 - 44 | 34.3% |
| D | 36 - 28 | 13.9% |
| F | 20 - | 4.1% |

$$\frac{0}{0}, \quad \frac{\infty}{\infty}$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

$$0 \cdot \infty$$

$$\lim_{x \to a} f(x) \cdot g(x) \qquad f(x) \to 0, \quad g(x) \to \infty$$

- **indeterminate products** $0 \cdot \infty$

$$\lim_{x\to a} f(x)\,g(x) = \lim_{x\to a} \frac{f(x)}{\frac{1}{g(x)}} = \lim_{x\to a} \frac{f'(x)}{\left(\frac{1}{g(x)}\right)'}$$

($f(x) \to 0$, $g(x) \to \infty$; $\frac{1}{g(x)} \to 0$)

Ex. 6 $\lim_{x\to 0^+} x \ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = -\lim_{x\to 0^+} \frac{x^2}{x} = 0$

($x \to 0^+$, $\ln x \to \infty$; $\frac{1}{x} \to \infty$)

- **indeterminate differences** $\infty - \infty \to \frac{0}{0}$ or $\frac{\infty}{\infty}$

Ex. 7 $\lim_{x\to(\frac{\pi}{2})^-} (\sec x - \tan x) = \lim_{x\to(\frac{\pi}{2})^-} \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)$

($\sec x \to \infty$, $\tan x \to \infty$)

$$= \lim_{x\to(\frac{\pi}{2})^-} \frac{1-\sin x}{\cos x} = \lim_{x\to(\frac{\pi}{2})^-} \frac{-\cos x}{-\sin x} = 0$$

($1 - \sin x \to 0$, $\cos x \to 0$)

- **indeterminate powers**

$$\lim_{x\to a}\left[f(x)\right]^{g(x)} \quad : \quad 0^0,\ \infty^0,\ \text{or } 1^\infty$$

$$= \lim_{x\to a} e^{\ln f(x)^{g(x)}} = \lim_{x\to a} e^{g(x)\ln f(x)}$$

$0^a = 0$, $a^0 = 1$

$1^a = 1$

$$\lim_{x\to a} e^{f(x)} = e^{\lim_{x\to a} f(x)}$$

Ex. 8 $\lim_{x\to 0^-}(1+\sin 4x)^{\cot x} = \lim_{x\to 0^-} e^{\cot x \ln(1+\sin 4x)} = e^{4}$

($\cot x \to \infty$, $\ln(1+\sin 4x) \to 0$)

$\lim_{x\to 0^+} \cot x \ln(1+\sin 4x) = \lim_{x\to 0^-} \dfrac{\cos x \ln(1+\sin 4x)}{\sin x}$ ($\sin x \to 0$, $\ln(1+\sin 4x) \to 0$)

$\overset{?}{=} \lim_{x\to 0^-} \cos x \lim_{x\to 0^-} \dfrac{\ln(1+\sin 4x)}{\sin x}$ ($\sin x \to 0$)

$\cos x \to 1$

$= \lim_{x\to 0^-} \dfrac{4\cos 4x}{1+\sin 4x}$ ($4\cos 4x \to 4$, $1+\sin 4x \to 1$)

$= 4$

$\infty \cdot 0 \to \frac{0}{0}$ or $\frac{\infty}{\infty}$

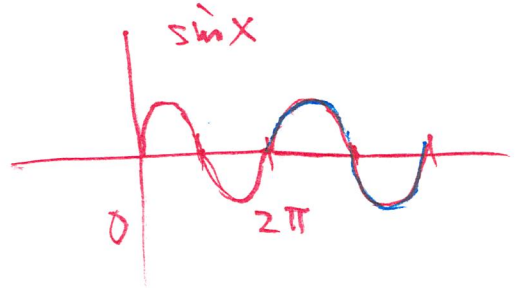
Ex. 9 $\lim_{x\to 0^+} x^x = \lim_{x\to 0^+} e^{x\ln x} = e^0 = 1$

$\lim_{x\to 0^+} x\ln x = 0$

($x \to 0$, $\ln x \to -\infty$)

# §4.5 Summary of Curve Sketching

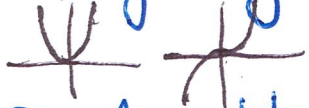
- **Guidelines for Sketching a Curve** $y = f(x)$




A. Domain

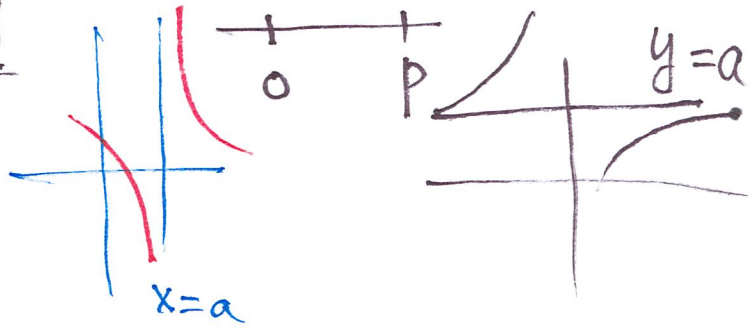
B. Intercepts $x$-intercepts $0 = f(x)$, $y$-intercepts $y = f(0)$

C. Symmetry



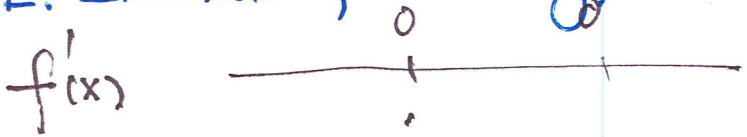
even $f(-x) = f(x)$, odd $f(-x) = -f(x)$, $f(x+p) = f(x)$

D. Asymptotes horizontal asymp: $\lim_{x \to \pm\infty} f(x) = a = y$




vertical asymp: $\lim_{x \to ?} f(x) = \pm\infty$ $x = ?$

E. Intervals of Increasing/Decreasing




F. Local Maximum and Minimum Values

G. Concavity and Points of Inflection $f''(x)$

**Ex. 1** Sketch the curve $y = \dfrac{2x^2}{x^2-1}$.

**A. Domain** $D = \{x \mid x^2 - 1 \neq 0\} = (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

$\Updownarrow$

$x \neq \pm 1$


**B. Intercepts** $y = f(0) = 0$

$0 = f(x) = \dfrac{2x^2}{x^2-1} \Rightarrow 2x^2 = 0 \Rightarrow x = 0$ $\quad (0, 0)$

**C. sym** even $\quad f(-x) = f(x)$ $\quad (0,1) \cup (1, \infty)$

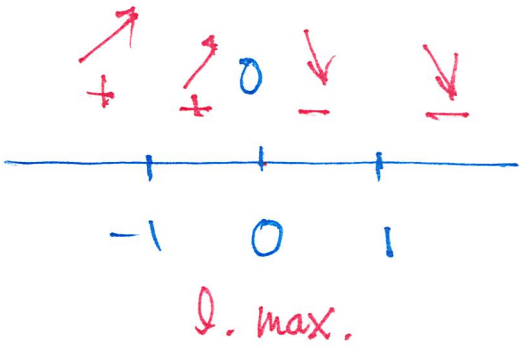
**D. Asymp** $\lim\limits_{x \to \pm\infty} \dfrac{2x^2}{x^2-1} = 2 = y$ hor. asy.

$\lim\limits_{x \to \pm 1} \dfrac{2x^2}{x^2-1} = \pm\infty \Rightarrow$ $x = -1$ and $x = 1$ v. asy.

$x \to \pm 1$, $x^2 - 1 \to 0$

**E.** $f'(x) = 2\dfrac{2x(x^2-1) - x^2(2x)}{(x^2-1)^2} = 2\dfrac{-2x}{(x^2-1)^2} = 0$

$\Rightarrow x = 0$ critical number




F.

6. $f''(x) = -4\left(\frac{x}{(x^2-1)^2}\right)' = -4\,\frac{(x^2-1)^2 - x\cdot 2(x^2-1)\cdot 2x}{(x^2-1)^4}$

$= -4\,\frac{x^2-1-(4x^2)}{(x^2-1)^3} = 4\,\frac{3x^2+1}{(x^2-1)^3} > 0$

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$(x^2-1)^3 > 0 \iff x^2-1>0 \iff x^2>1 \Rightarrow x<-1$ or $x>1$