Ex. 2 Sketch the graph of $f(x) = \dfrac{x^2}{\sqrt{x+1}}$.

A. Domain $= \{x \mid x+1>0\} = \{x \mid x > -1\} = (-1, +\infty)$

B. x- and y- intercepts are $f(x)=0 = \dfrac{x^2}{\sqrt{x+1}} \Rightarrow x=0$, $y=f(0)=0$

C. symmetry none

D. $\lim\limits_{x\to\infty} \dfrac{x^2}{\sqrt{x+1}} = \lim\limits_{x\to\infty} \dfrac{x^2}{\sqrt{x}} = \infty$, no hor. asy.

$\lim\limits_{x\to -1^+?} \dfrac{x^2}{\sqrt{x+1}} = \infty \Rightarrow x=-1$ V. asy.

E. $f'(x) = \dfrac{x(3x+4)}{2(x+1)^{3/2}}$

$f'(x)=0 = \dfrac{x(3x+4)}{2(x+1)^{3/2}} \Leftrightarrow x(3x+4)=0$

$\Leftrightarrow \boxed{x=0}$ or $x = -\dfrac{4}{3} < -1$

F. $f'(x)$: $-$ on $(-1, 0)$, $+$ on $(0, \infty)$

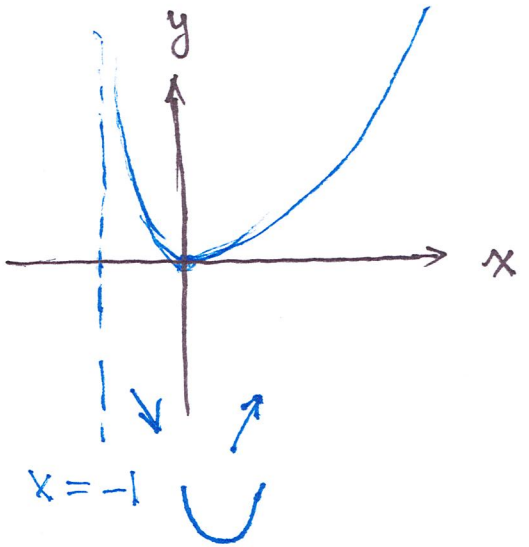
$-1 \downarrow 0 \uparrow$

$f(0) = 0$ local min

G. $f''(x) = \dfrac{3x^2+8x+8}{4(x+1)^{5/2}} > 0$ $\cup$

$b^2 - 4ac =$

$= -8 \pm \sqrt{8^2 - 4\cdot 3 \cdot 8}$ / 6



Ex. 3 Sketch the graph of $f(x) = xe^x$.

A. Domain $= (-\infty, +\infty)$

B. x- and y- intercepts are $y = f(0) = 0$, $xe^x = 0 \Rightarrow x = 0$

C. symmetry none

D. $\lim_{x\to\infty} xe^x = \infty$

hor. asy.

$e^{-\infty} = \frac{1}{e^{\infty}} \to 0$

$\lim_{x\to-\infty} xe^x = \lim_{x\to-\infty} \frac{x}{e^{-x}} = \lim_{x\to-\infty} \frac{1}{-e^{-x}} = \boxed{0 = y}$ hor. asy.

$\lim_{x\to-\infty} \frac{e^x}{\frac{1}{x}} = \lim_{x\to-\infty} \frac{e^x}{-\frac{1}{x^2}}$

$\lim_{x\to} xe^x \neq \infty$?

no ver. asy

$f(x)$	$-$	0	$+$
	$\searrow$	-1	$\nearrow$

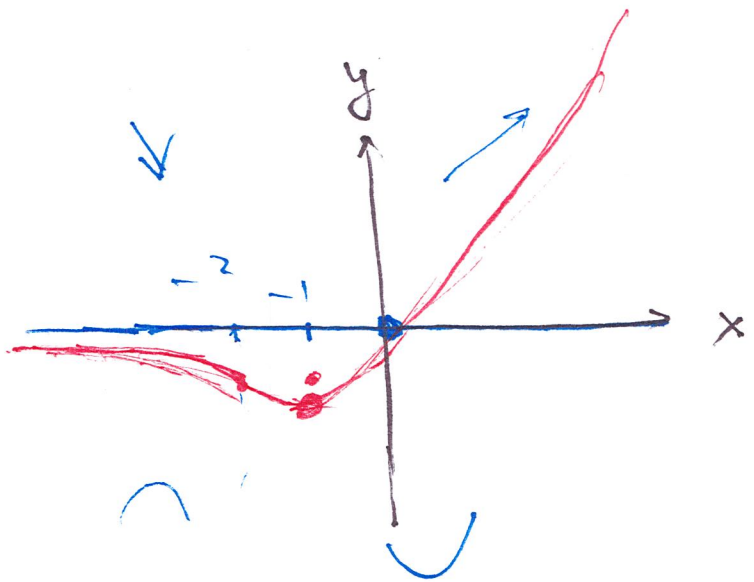
$f(-1) = -\frac{1}{e}$ l. min

E. $f'(x) = (x+1)e^x$

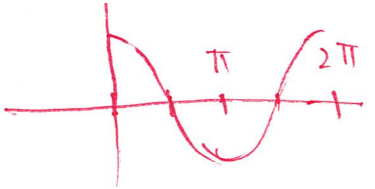
F. $f'(x) = 0 \Rightarrow 0 = (x+1)e^x \Rightarrow x = -1$

G. $f''(x) = (x+2)e^x = 0 \Rightarrow x = -2$

$f''(x)$	$-$	0	$+$
	$\cap$	-2 P.I	$\cup$



Ex. 4 Sketch the graph of $f(x)=\dfrac{\cos x}{2+\sin x}$, $2+\sin x>0$



A. Domain $= R$

B. y-intercept $y=f(0)=\frac{1}{2}$ x-intercept $\cos x=0 \Rightarrow x=\frac{\pi}{2}\pm 2n\pi$, $\frac{3\pi}{2}\pm 2n\pi$

C. symmetry. $f(x+2\pi)=\dfrac{\cos(x+2\pi)}{2+\sin(x+2\pi)}=\dfrac{\cos x}{2+\sin x}$

even $f(-x)=f(x)$

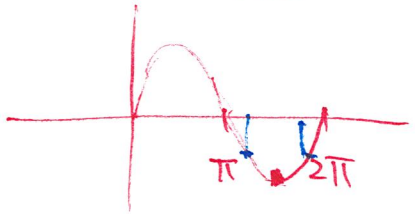
odd $f(-x)=-f(x)$

D. Asymptotes $\lim_{x\to\infty}\dfrac{\cos x}{2+\sin x}\neq$ None

$f\left(\frac{7\pi}{6}\right)=\dfrac{-\frac{\sqrt3}{2}}{2+\left(-\frac{1}{2}\right)}=-\frac{\sqrt3}{3}$

$f\left(\frac{11\pi}{6}\right)=\dfrac{\sqrt3/2}{2-\frac{1}{2}}=\frac{\sqrt3}{3}$

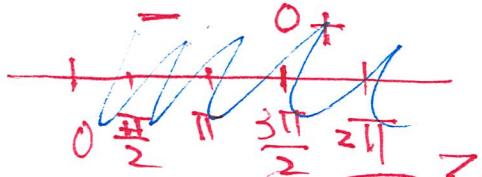
E. $f'(x)=-\dfrac{2\sin x+1}{(2+\sin x)^2}>0$



$0=2\sin x+1 \Rightarrow \sin x=-\frac{1}{2} \Rightarrow x=\frac{7\pi}{6}, \frac{11\pi}{6}$

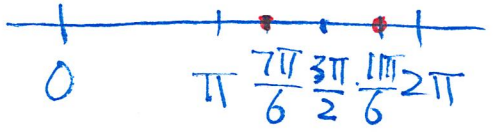
$f'(x)$

F.



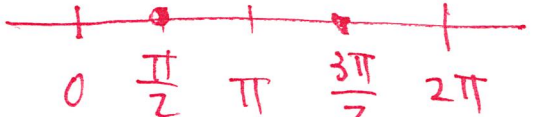
>0

$f'(x)$ $-\ 0\ +\ 0\ -$



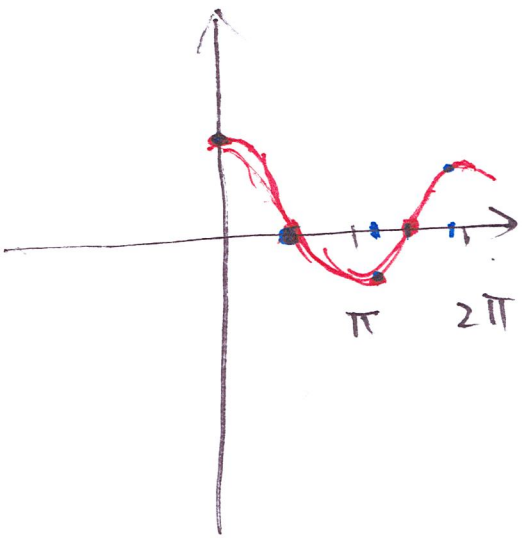
G. $f''(x)=-\dfrac{2\cos x(1-\sin x)}{(2+\sin x)^3}>0$

$f''(x)$ l.min l.max $\cap\ 0\ \cup\ +\ 0\ \cap\ -$



$f''(x)=0$ $\cos x(1-\sin x)=0$

$\Leftrightarrow \cos x=0$ or $\sin x=1 \Leftrightarrow x=\frac{\pi}{2}, \frac{3\pi}{2}$



Ex. 5 Sketch the graph of $y = \ln(4 - x^2)$.

A. domain $= \{x \mid 4 - x^2 > 0\} = \{x \mid x^2 < 4\} = \{x \mid |x| < 2\} = (-2, 2)$

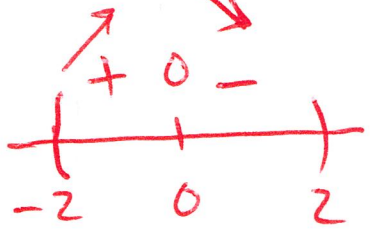
B. y-intercept $y = f(0) = \ln 4$ x-intercept: $\ln(4 - x^2) = 0 \Rightarrow 4 - x^2 = 1 \Rightarrow x = \pm\sqrt{3}$ ($\ln 1 = 0$)

C. symmetry $f(-x) = f(x)$ even

D. $\lim_{x \to 2^-} \ln(4 - x^2) = -\infty$ $x = 2$ ver. asy.

$\lim_{x \to -2^+} \ln(4 - x^2) = -\infty$ $x = -2$

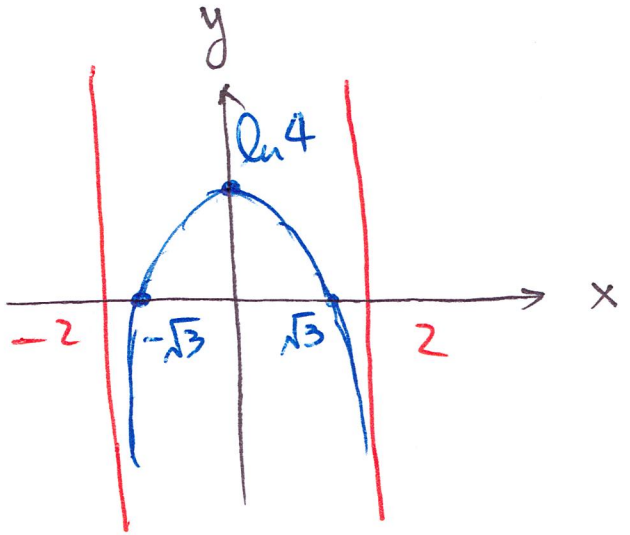
E. $f'(x) = \dfrac{-2x}{4 - x^2} = 0 \Rightarrow x = 0$



F. $f(0) = \ln 4$ l. max

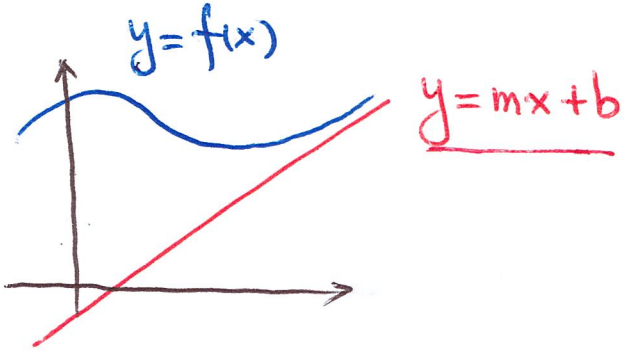
G. $f''(x) = -\dfrac{8 + 2x^2}{(4 - x^2)^2} < 0$

∩



• slant asymptotes $y = mx + b$

$$\lim_{x\to\infty}\left[f(x) - (mx+b)\right] = 0$$



Ex. 6 Sketch the graph of $f(x) = \dfrac{x^3}{x^2+1}$

$x^2+1 \mid x^3 \quad x$

$\to x^3 + x$

$-x$

$= x - \dfrac{x}{x^2+1}$

$\Rightarrow \lim_{x\to\infty}\left[f(x) - x\right] = \lim_{x\to\infty} -\dfrac{x}{x^2+1} = 0$

$y = x$ slant asy.

A. Domain = $\mathbb{R}$

B. x-intercept and y-intercept are

$x^3 = 0 \Rightarrow x = 0$

$y = f(0) = 0$

C. sym

D. slant asy. $y = x$

$$f(x) = \frac{x^2(x^2+3)}{(x^2+1)^2} \to 0$$

$f'(x) = 0 \iff x^2 = 0 \iff x = 0$

		0	
f'	+	0	+
f	↗	0	↗

$f(\sqrt{3}) = \frac{3\sqrt{3}}{4}$

E. $f'(x) = \dfrac{x^2(x^2+3)}{(x^2+1)^2} = 0 \iff x^2 = 0 \iff x = 0$

F. no l. min or max

		$-\sqrt{3}$		0		$\sqrt{3}$	
f''	+	0	−	0	+	0	−
f	∪		∩		∪		∩

G. $f''(x) = \dfrac{2x(3-x^2)}{(x^2+1)^3} = 0 \iff x = 0$ or $x = \pm\sqrt{3}$