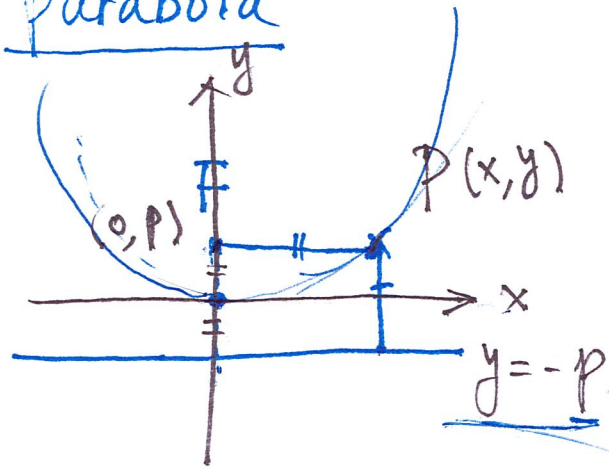


# parabola



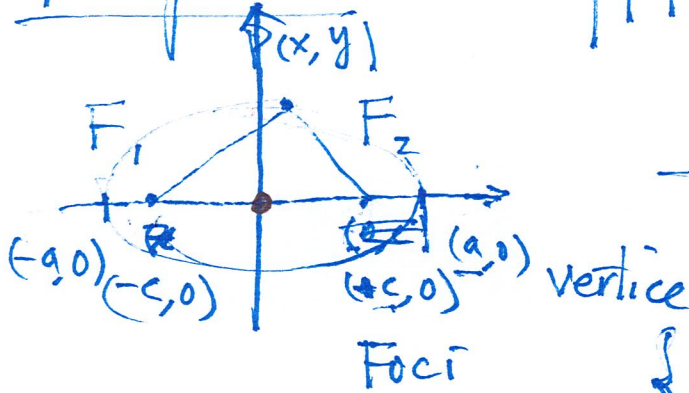
$$|PF| = |y+p|$$

$$x^2 = 4py$$

focus  $(0, p)$

$y = -p$  — directrix

# Ellipse



$$|PF_1| + |PF_2| = 2a = \text{constant}$$

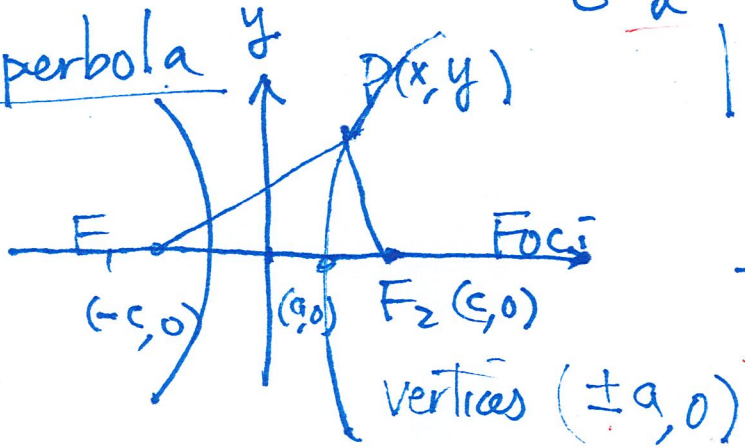
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2 = a^2 - c^2$$

$$\begin{cases} \frac{x}{a} + \frac{y}{b} = 0 \\ \frac{x}{a} - \frac{y}{b} = 0 \end{cases}$$

$$\left(\frac{x}{a} + \frac{y}{b}\right) \left(\frac{x}{a} - \frac{y}{b}\right) = 1$$

# Hyperbola



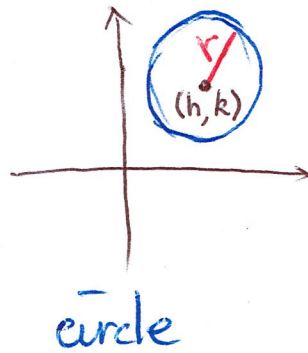
$$|PF_1| - |PF_2| = \pm 2a = \text{const}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

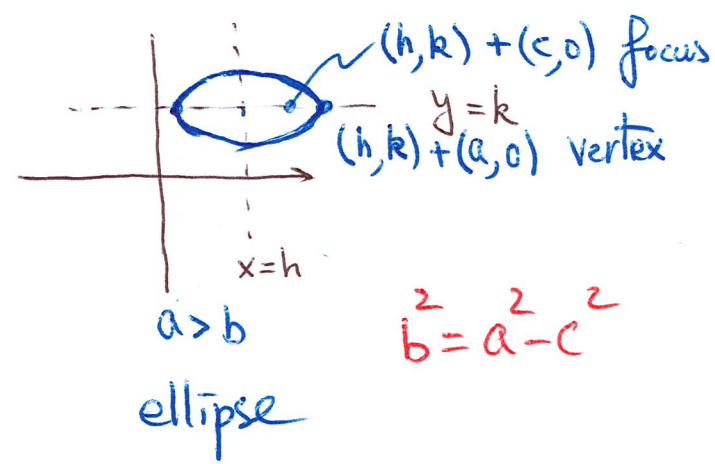
$$c^2 = a^2 + b^2$$

# Shifted Conics

circle  $(x-h)^2 + (y-k)^2 = r^2$



ellipse  $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$



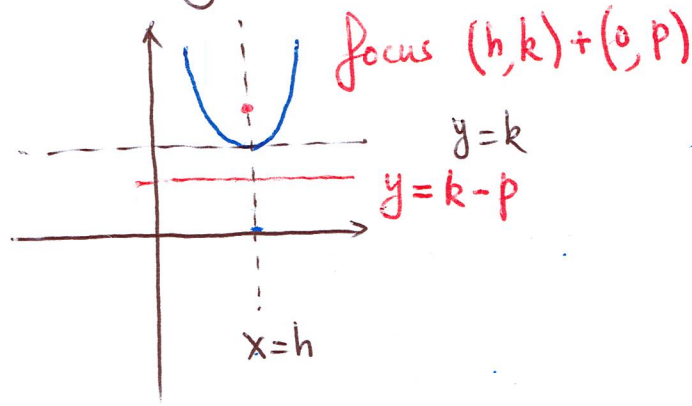
parabola  $y - k = a(x - h)^2$

$a = \frac{1}{4p}$

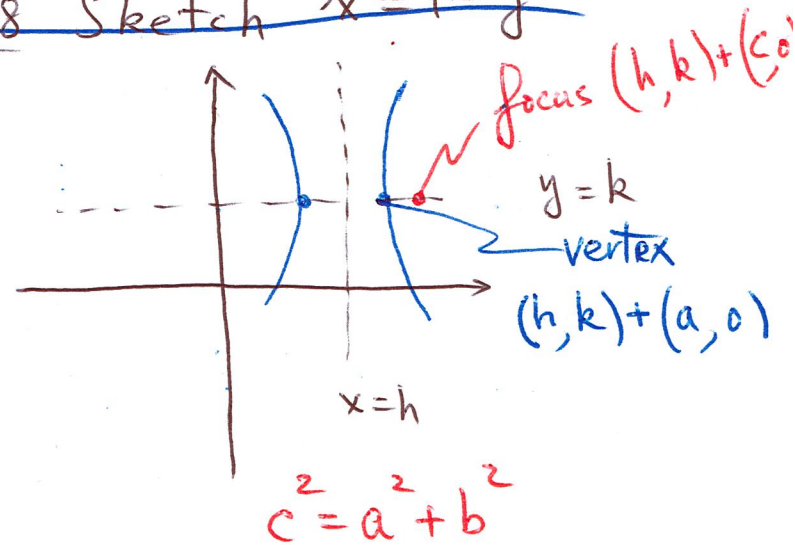
$(x-h)^2 = 4p(y-k)$

hyperbola  $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

~~Ex. 7 Sketch  $y = 2x^2 - 4x + 1$~~



~~Ex. 8 Sketch  $x = 1 - y^2$~~



• shifted conics replacing  $x$  and  $y$  by  $x-h$  and  $y-k$

Ex. 6 Find an equation of the ellipses with foci  $(2, -2), (4, -2)$  and vertices  $(1, -2), (5, -2)$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

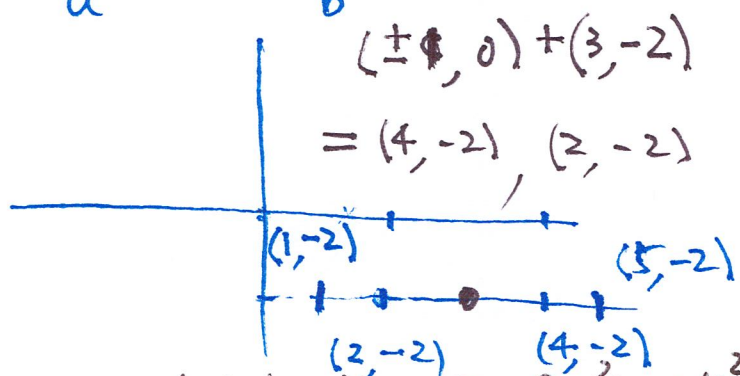
$$(h, k) = (3, -2)$$

$$\frac{(x-3)^2}{2^2} + \frac{(y+2)^2}{3} = 1$$

$$c = 1$$

$$a = 2$$

$$b^2 = a^2 - c^2 = 4 - 1 \Rightarrow b = \sqrt{3}$$



Ex. 7 Sketch the conic  $9x^2 - 4y^2 - 72x + 8y + 176 = 0$  and find its foci.

$$0 = 9x^2 - 72x - (4y^2 - 8y) + 176 = 9(x^2 - 8x + 4^2) - 4(y^2 - 2y + 1)$$

$$= 9(x-4)^2 - 4(y-1)^2 + 36 - 4 + 176$$

$$\begin{array}{r} -9 \times 4^2 + 4 + 176 \\ \hline \frac{16}{9} \\ -144 \\ + 176 \\ \hline 32 \end{array}$$

$$4(y-1)^2 - 9(x-4)^2 = 36$$

$$(4, 1), a = 3, b = 2$$

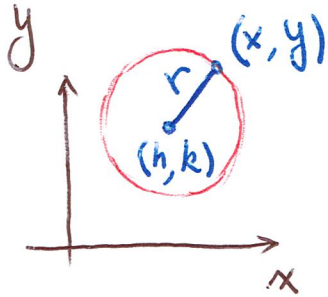
$$\frac{(y-1)^2}{3^2} - \frac{(x-4)^2}{2^2} = 1$$

$$(0, \pm\sqrt{13}) + (4, 1) c = \sqrt{9 + 4} = \sqrt{13}$$

$$= (4, 1 \pm \sqrt{13})$$

# Appendix C Graphs of Second-Degree Equations

• Circles  $(x-h)^2 + (y-k)^2 = r^2$



Ex. 1 Find an equation of the circle with radius 3 and center  $(2, -5)$ .

$$(x-2)^2 + (y+5)^2 = 9$$

Ex. 2 Sketch the graph of the equation  $x^2 + y^2 + 2x - 6y + 7 = 0$  by first showing that it represents a circle and then finding its center and radius.

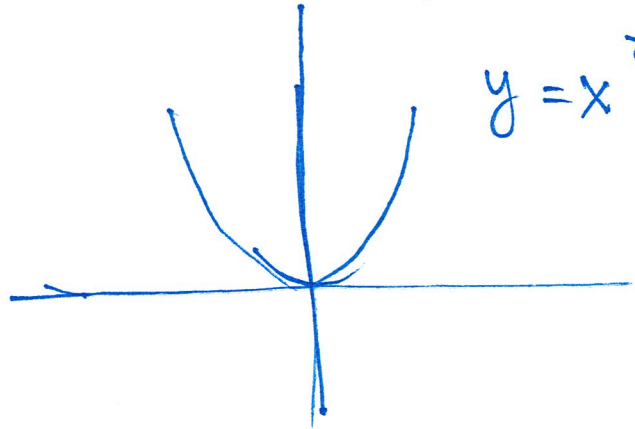
$$\begin{aligned} 0 &= (x^2 + 2x + 1) - 1 + (y^2 - 6y + 3^2) - 3^2 + 7 \\ &= (x+1)^2 + (y-3)^2 - 3 \end{aligned}$$

$$(-1, 3), \quad r = \sqrt{3}$$

• parabolas

$$\underline{y = ax^2 + bx + c}$$

Ex. 3 Draw the graph of  $y = x^2$ .



$$y = x^2$$

even  $y(-x) = (-x)^2 = x^2 = y(x)$

sym. w.r.t.  $y$ -axis.

Ex. 4 Sketch the region bounded by the parabola  $x = y^2$  and the line  $y = x - 2$ .