8

Given $f'(2)=5$, determine

(i) $g'(1)$ where $g(x)=f(2x)$

$$g'(x)\Big|_{x=1} = f'(2x)\cdot 2\Big|_{x=1} = f'(2)\cdot 2 = 5\cdot 2 = 10$$

$$(2+4h)-2 = 4h$$

(ii) $$\lim_{h\to 0} \frac{f(2+4h)-f(2)}{3h}\cdot\frac{4h}{4h} = \frac{4}{3}\lim_{h\to 0}\frac{f(2+4h)-f(2)}{4h}$$

$$= \frac{4}{3}\cdot f'(2) = \frac{4}{3}\times 5 = \frac{20}{3}$$

$$(2+4h)-(2+5h) = -h$$

(iii) $$\lim_{h\to 0} \frac{f(2+4h)-f(2+5h)}{9\cancel{h}}\cdot\frac{-\cancel{h}}{-h}$$

$$= -\frac{1}{9}\lim_{h\to 0}\frac{f(2+4h)-f(2+5h)}{-h} = -\frac{1}{9}\cdot f'(2) = -\frac{5}{9}$$

#9 Find the values a and b so that

$$f(x) = \begin{cases} x^2 - a, & \text{if } x \le 1 \\ \dfrac{3x^2+12x-b}{x^2+2x-3} & \text{if } x > 1 \end{cases}$$ is cont. on $(-\infty, \infty)$.

$$\frac{x^2+2x-3}{} = \frac{3x^2+12x-b}{(x+3)(x-1)}$$

$x = 1$ or -3

$\underline{x=1}$

$$\lim_{x \to 1^-} (x^2 - a) = 1 - a$$

$$\lim_{x \to 1^+} \frac{3x^2+12x-b}{(x+3)(x-1)} \text{ has a limit } \Longrightarrow \boxed{\left[3x^2+12x-b\right]_{x=1} = 0}$$

$3 + 12 - b = 0 \Rightarrow b = 15$

$$= \lim_{x \to 1^+} \frac{3\left[x^2+4x-5\right]}{(x+3)\cancel{(x-1)}} = 3\frac{(x+5)\cancel{(x-1)}}{} = 3 \cdot \frac{6}{4} = \frac{9}{2}$$

$$\boxed{1 - a = \frac{9}{2}} \Longrightarrow a = -\frac{7}{2}$$

#11

$$\lim_{x\to 0} \frac{3\sin(3x)}{3(5x)} \overset{y=3x}{=\!=} \lim_{y\to 0} \frac{\sin y}{\frac{5}{3}y} = \frac{3}{5}$$

$$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$$

$$\lim_{x\to\infty} \frac{\sin(3x)}{5x} = 0$$

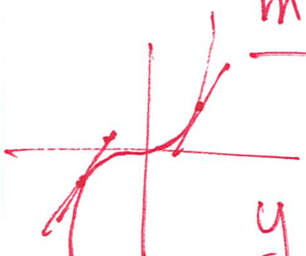
$$\left|\frac{\sin 3x}{5x}\right| \le \left|\frac{1}{5x}\right| = \frac{1}{5|x|} \Rightarrow 0$$

$$\lim_{x\to 0} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = \lim_{x\to 0} x \sin\left(\frac{1}{x}\right) = 0$$

#12 Find the eq. of the line tangent to $y=x^3$, parallel to $y=3x$

$m = 3$

$y'(x) = (x^3)' = 3x^2 = 3 \Rightarrow x^2 = 1$



$\Rightarrow x = -1$, or 1 $\quad (-1,-1)$, $(1,1)$

$y = -1$ or 1

$y+1 = 3(x+1)$

or

$y-1 = 3(x-1)$

Spring 2016

Questions

1. Find the exact value of each expression.

(i) $e^{\ln(\ln e^3)}$ $= \ln e^3 = 3$

(ii) $\sin^{-1}\left(\sin\frac{7\pi}{5}\right)$ $= -\sin^{-1}\sin\frac{2}{5}\pi$

HINT: The range of the function "$\sin^{-1}$" is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$= -\frac{2}{5}\pi$

A. (i) e^3 (ii) $\frac{7\pi}{5}$

B. (i) e^3 (ii) $-\frac{2\pi}{5}$

C. (i) 3 (ii) $\frac{7\pi}{5}$

D. (i) 3 (ii) $\frac{2\pi}{5}$

E. (i) 3 (ii) $-\frac{2\pi}{5}$

$$\frac{7\pi}{5} = \frac{5\pi + 2\pi}{5} = \pi + \frac{2}{5}\pi$$

$$\sin\left(\frac{7\pi}{5}\right) = \sin\left(\pi + \frac{2}{5}\pi\right)$$

$$= \sin\pi \cos\frac{2}{5}\pi + \cos\pi \sin\frac{2}{5}\pi$$

$$= -\sin\frac{2}{5}\pi$$

3. We have the information

$$\cos\theta = -\frac{12}{13} \quad \text{and} \quad \frac{\pi}{2} < \theta < \pi.$$

Determine the value of $\tan\theta$.

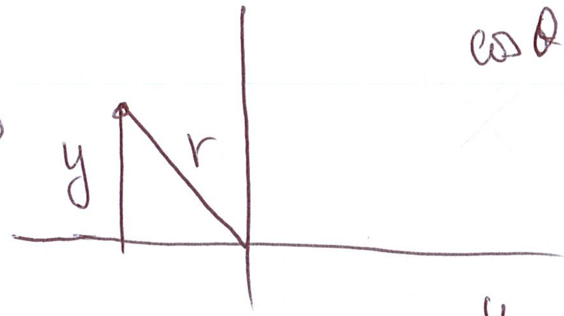
A. $\frac{12}{5}$

B. $-\frac{12}{5}$

C. $\frac{5}{12}$

D. $-\frac{5}{12}$

E. $-\frac{5}{13}$



$$\cos\theta = \frac{x}{r} = -\frac{12}{13}$$

$$x = -12, \quad r = 13$$

$$y = \sqrt{r^2 - x^2}$$

$$= \sqrt{(13)^2 - (12)^2}$$

$$= \sqrt{(13+12)(13-12)}$$

$$= \sqrt{25} = 5$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{x}{y}$$

$$= \frac{-12}{5}$$

5. Find the value of $f'(1)$ for the function $f(x) = \frac{4\sqrt{x}}{x^2 - 2}$.

A. 10

B. -10

C. 0

D. 1

E. -1

$$f'(x) = 4\,\frac{\frac{1}{2}x^{-\frac{1}{2}}\cdot(x^2-2) - \sqrt{x}\cdot 2x}{(x^2-2)^2}$$

$$f'(1) = 4\,\frac{\frac{1}{2}\cdot 1\cdot(1-2) - 1\cdot 2\cdot 1}{(1^2-2)^2}$$

$$= 4\cdot\left[-\frac{1}{2} - 2\right] = -4\left[\frac{1}{2}+2\right]$$

$$= -4\,\frac{1+4}{2} = -2\cdot(5) = -10$$

$$\sqrt{x^2} = |x|$$

7. Compute the following limits:

(i) $\lim_{x\to-\infty} \frac{\sqrt{3x^2+6}}{5-2x}$ $= \lim_{x\to-\infty} \frac{\sqrt{3}\,|x|}{-2x}$

(ii) $\lim_{x\to\infty} \left(\sqrt{x^2+x}-x\right)$

A. (i) $\frac{\infty}{\infty} = 1$ (ii) $\infty - \infty = 0$

B. (i) $\frac{\sqrt{3}}{2}$ (ii) $\frac{1}{2}$

C. (i) $-\frac{\sqrt{3}}{2}$ (ii) $\frac{1}{2}$

D. (i) $\frac{\sqrt{3}}{2}$ (ii) 0

E. (i) $-\frac{\sqrt{3}}{2}$ (ii) 0

$$= \lim_{x\to-\infty} \frac{\sqrt{3}(-x)}{-2x}$$

$$= \frac{\sqrt{3}}{2}$$

$$\lim_{x\to\infty} \left(\sqrt{x^2+x}-x\right) \cdot \frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x}$$

$$= \lim_{x\to\infty} \frac{(\cancel{x^2}+x)-\cancel{x^2}}{\sqrt{x^2+x}+x} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2+x}+x}$$

$$x$$

$$= \lim_{x\to\infty} \frac{x}{x+x} = \frac{1}{2}$$

9. Choose the right description of the interval(s) where the function

$$f(x) = |(x+2)(x-1)^2| = |x+2|\,|(x-1)^2|$$

is

$$= |x+2|(x-1)^2$$

(i) continuous,
(ii) differentiable.

A. (i) $(-\infty, \infty)$, (ii) $(-\infty, \infty)$
B. (i) $(-\infty, -2) \cup (1, \infty)$, (ii) $(-2, 1)$
C. (i) $(-\infty, \infty)$, (ii) $(-\infty, -2) \cup (-2, \infty)$
D. (i) $(-\infty, \infty)$, (ii) $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$
E. (i) $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$ (ii) $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$

$$= \begin{cases} (x+2)(x-1)^2 & x+2>0 \\ -(x+2)(x-1)^2 & x+2<0 \end{cases}$$

$x = -2$

$(-\infty, -2) \cup (-2, \infty)$

11. Find the equation(s) of

(i) the horizontal asymptote(s)
(ii) the vertical asymptote(s)

$(x+3)(x-1)$

for the graph of the function

$$y = f(x) = \frac{5x^2 + 5x - 30}{x^2 + 2x - 3}. = \frac{5\cancel{(x+3)}(x-2)}{\cancel{(x+3)}(x-1)}$$

A. (i) $y = -5$ (ii) $x = 1$

B. (i) $y = 5$ (ii) $x = 1$

C. (i) $y = 5$ (ii) $x = -3$ and $x = 1$

D. (i) $y = 5$ and $y = -5$ (ii) $x = -3$ and $x = 1$

E. The graph has the slant asymptote $y = 5x - 10$, but neither horizontal nor vertical asymptote.

$\lim_{x \to 1^-} f(x) = \infty$, $\lim_{x \to 1^+} f(x) = -\infty$ $x = 1$

V. a.

$\lim_{x \to -3} f(x) = \frac{5 \cdot 5}{4} = \frac{25}{4}$

$\lim_{x \to \infty} f(x) = 5$

$y = 5$

$\lim_{x \to -\infty} f(x) = 5$