2. Consider the following function defined over the domain $(-\infty, 4) \cup (4, \infty)$

$$y = f(x) = \frac{3x+2}{x-4}.$$

Choose the right answer for

(i) the formula for the inverse function $f^{-1}(x)$,
(ii) the range for the inverse function.

A. The function $y = f(x)$ is not one-to-one over the specified domain, and hence its inverse function does not exist.

B. (i) $f^{-1}(x) = \dfrac{x-3}{3x+2}$ (ii) $(-\infty, 4) \cup (4, \infty)$

C. (i) $f^{-1}(x) = \dfrac{x-3}{3x+2}$ (ii) $(-\infty, 3) \cup (3, \infty)$

D. (i) $f^{-1}(x) = \dfrac{4x+2}{x-3}$ (ii) $(-\infty, 4) \cup (4, \infty)$

E. (i) $f^{-1}(x) = \dfrac{4x+2}{x-3}$ (ii) $(-\infty, 3) \cup (3, \infty)$

$$y = \frac{3x+2}{x-4} \Rightarrow xy - 4y = 3x + 2$$

$$\Rightarrow (y-3)x = 4y + 2 \Rightarrow x = \frac{4y+2}{y-3}$$

$$y = \frac{4x+2}{x-3}$$

range f^{-1} = domain f

4. We want to compute the following limit

$$\lim_{x\to\infty} \frac{\sin x}{x}.$$

Choose the right answer with correct reasoning.

A. $\displaystyle\lim_{x\to\infty} \frac{\sin x}{x}$ is one of the basic limits involving the trigonometric functions, and it is well-known to be equal to $\cancel{1}$.

B. Since $-1 \le \sin x \le 1$, we have $\displaystyle -\frac{1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$ when x is a positive number. By observing both $\displaystyle -\frac{1}{x}$ and $\displaystyle \frac{1}{x}$ go to 0 as x approaches ∞ and by applying the Squeeze Theorem, we conclude that the limit we want to compute is equal to 0.

C. For a positive number a, we have $\displaystyle \frac{1}{a}\cdot \sin x = \sin\left(\frac{1}{a}\cdot x\right)$. Applying this rule, we have $\displaystyle \lim_{x\to\infty} \frac{\sin x}{x} = \lim_{x\to\infty} \frac{1}{x}\cdot \sin x = \lim_{x\to\infty} \sin\left(\frac{1}{x}\cdot x\right) \cancel{=} \sin 1.$

D. $\displaystyle \lim_{x\to\infty} \frac{\sin x}{x} \cancel{=} \left[\lim_{x\to\infty} \sin x\right]\cdot\left[\lim_{x\to\infty} \frac{1}{x}\right]$. Since $\sin x$ oscillates as x approaches ∞, $\displaystyle\lim_{x\to\infty} \sin x$ does not exist. Therefore, the limit we want to compute does not exist, either.

E. $\displaystyle \lim_{x\to\infty} \frac{\sin x}{x} \cancel{=} \left[\lim_{x\to\infty} \sin x\right]\cdot\left[\lim_{x\to\infty} \frac{1}{x}\right]$. Since $\displaystyle\frac{1}{x}$ goes to 0 as x approaches ∞, we conclude that $\displaystyle\lim_{x\to\infty} \frac{1}{x} = 0$. Therefore, the limit we want to compute is also equal to 0.

6. Mark asks the following question:

Is there a number c which is exactly 3 more than its cube c^3 ?

Nancy answers, saying "Yes. We can apply the Intermediate Value Theorem to the function $f(x) = (x^3 + 3) - x$ with $N = 0$."

Choose the interval where such a number c is guaranteed to exist as in Nancy's answer.

A. $(-4, -3)$

B. $(-3, -2)$

C. $(-2, -1)$

D. $(-1, 0)$

E. $(0, 1)$

$f(0) = 3 > 0$

$f(-1) = -1 + 3 + 1 = 3 > 0$

$f(-2) = -8 + 3 + 2 = -3 < 0$

8. How many solutions are there on the interval $[0, 2\pi]$ for the equation $4\cos x = \sin(2x)$?

A. 4

B. 3

C. 2

D. 1

E. 0

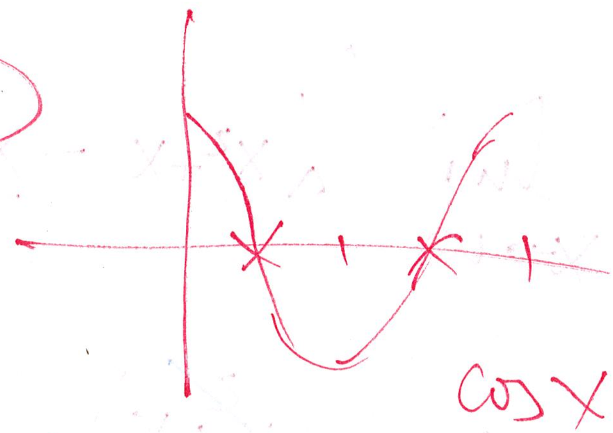
$$\overset{2}{\cancel{4}}\cos x = \underline{\sin(2x)} = \underline{\cancel{2}\sin x \cos x}$$

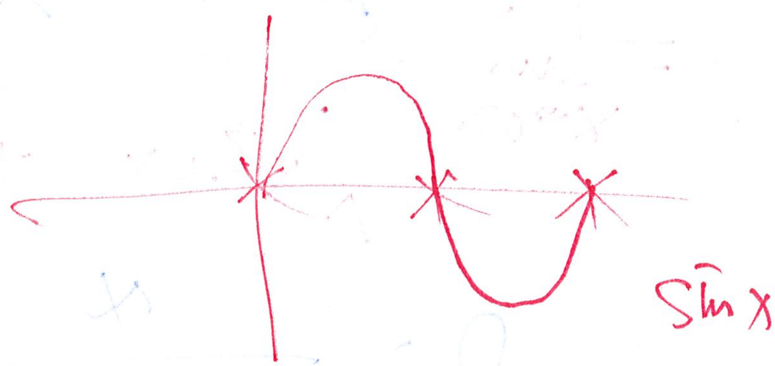
$$0 = 2\cos x - \sin x \cos x$$

$$= (2 - \sin x)\cos x$$

$$\Rightarrow 2 - \sin x \neq 0$$

or $\underline{\cos x = 0}$





10. Let $f(x)$ be the function defined as follows:

$$f(x) = \begin{cases} 1 & \text{if} \quad x < 0 \\ e^x & \text{if} \quad x \geq 0. \end{cases}$$

Compute the value of $f'(0)$ if it exists.

If $f'(0)$ does not exist, then write DNE.

A. 1

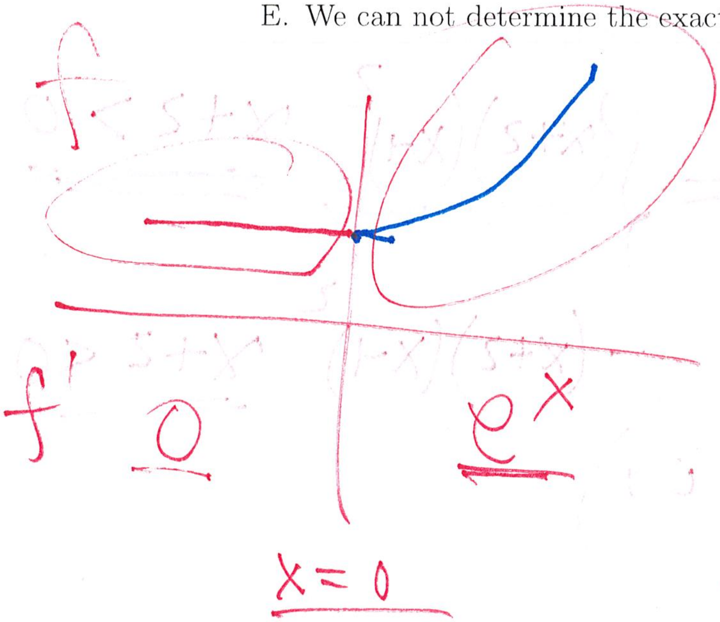
B. 2

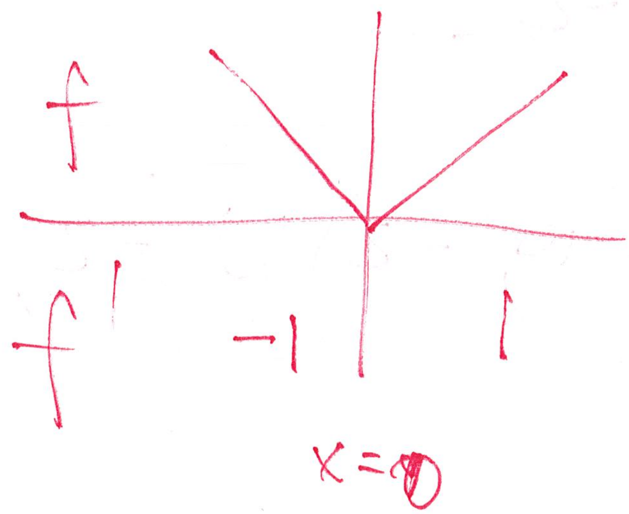
C. DNE

D. 0

E. We can not determine the exact value of $f'(0)$ from the given information.

$f(x) = |x| = \begin{cases} -x, & x<0 \\ x & x \geq 0 \end{cases}$





12. Find the values of a and b so that the function

$$f(x) = \begin{cases} \dfrac{x^2+x-a}{x+3} & \text{if } x < -3 \\ bx+1 & \text{if } x \geq -3 \end{cases}$$

is continuous on $(-\infty, \infty)$.

A. $a = 9, b = 2$

B. $a = 9, b = 1$

C. $a = 6, b = 2$

D. $a = 6, b = 1$

E. No matter how we choose the values for a and b, the function $f(x)$ can never be continuous on $(-\infty, \infty)$.

$$\left[x^2+x-a\right]_{x=-3} = 0 = 9-3-a = 6-a$$

$$\boxed{a=6}$$

$$\lim_{x\to -3^-} \frac{x^2+x-6}{x+3} = \frac{\cancel{(x+3)}(x-2)}{\cancel{x+3}} = -5$$

$$\lim_{x\to -3^+} (bx+1) = \cancel{3b} -3b+1$$

$$3b = 6 \Rightarrow b=2$$