

#1. P20 #6 $g(x) = \sqrt{16-x^4}$

$$2^4 - x^4 = 16 - x^4 \geq 0 \Rightarrow x^4 \leq 16$$

$$= \underbrace{(2^2 + x^2)}_{\geq 0} \underbrace{(2^2 - x^2)}_{\geq 0} \geq 0 \Rightarrow 2^2 - x^2 \geq 0$$

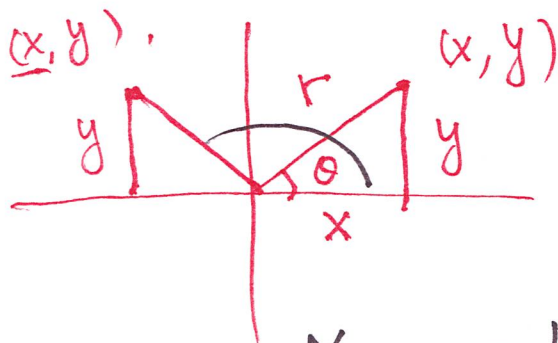
$$|x| \leq 2 \Rightarrow -2 \leq x \leq 2$$

$$2^4 = (2^2)^2 \quad x^4 = (x^2)^2$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$x^2 \leq 2^2$$

#2. $\sin \theta = \frac{3}{5}$, $\frac{\pi}{2} < \theta < \pi$



$$\sin \theta = \frac{y}{r} = \frac{3}{5}$$

$$\Rightarrow x = -4$$

$$\cos \theta = \frac{x}{r} = \frac{-4}{5}, \quad \tan \theta = \frac{\sin \theta}{\cos \theta} = -\frac{3}{4}$$

$$r = 5, \quad y = 3$$

$$x = \sqrt{r^2 - y^2}$$

$$= \sqrt{25 - 9} = \sqrt{16} = 4$$

#3. $x \in [0, 2\pi]$

$$\cos x = \cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - [1 - \cos^2 x]$$

$$\cos(x+x) = \cos x \cos x - \sin x \sin x$$

$$\sin^2 x + \cos^2 x = 1$$

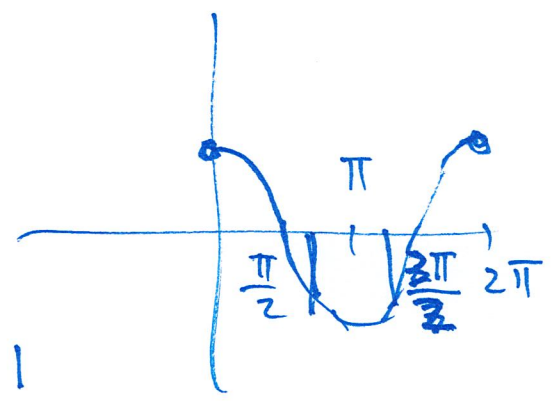
$$\cos x = 2 \cos^2 x - 1$$

$$\begin{aligned} \cos(A+B) &= \cos A \cos B - \sin A \sin B \\ \sin(A+B) &= \sin A \cos B + \sin B \cos A \end{aligned}$$

$$2y^2 - y - 1 = 0 \quad y = \cos x$$

$$y = \frac{1 \pm \sqrt{1^2 - 4 \cdot 2 \cdot (-1)}}{2 \cdot 2} = \frac{1 \pm \sqrt{9}}{4}$$

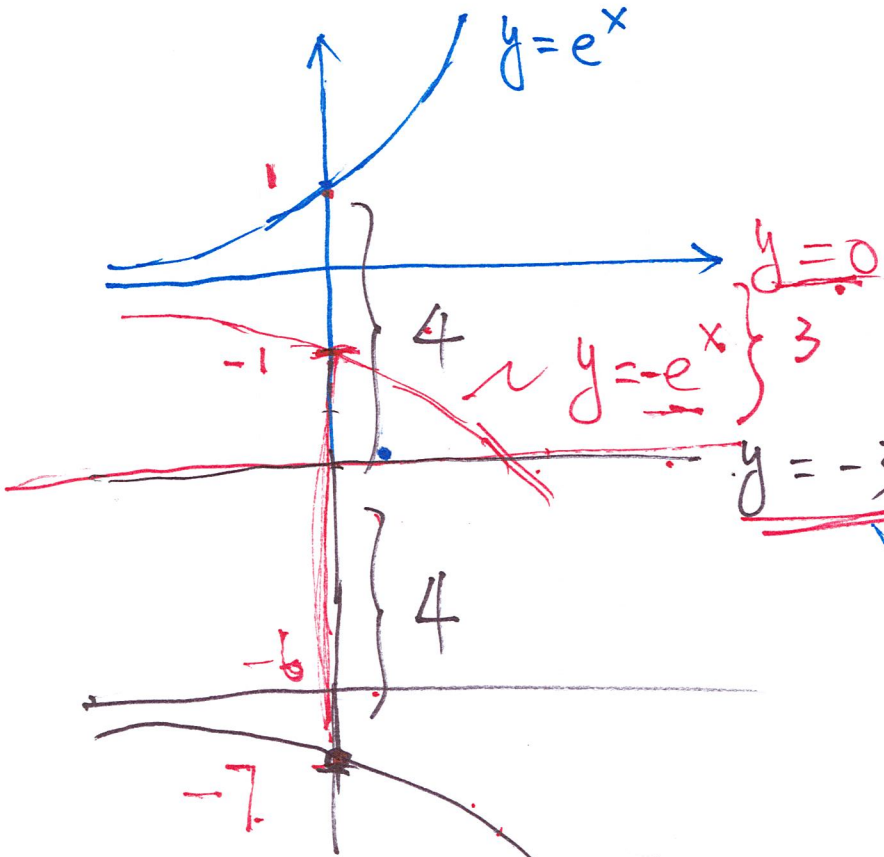
$$\cos x = \frac{1 - \sqrt{5}}{4} = -\frac{1}{2}, \quad \cos 2x = \frac{1 + \sqrt{5}}{4} = 1$$



$$\cos \theta = \frac{x}{r} \quad \begin{aligned} x &= -1 \\ r &= 2 \end{aligned}$$

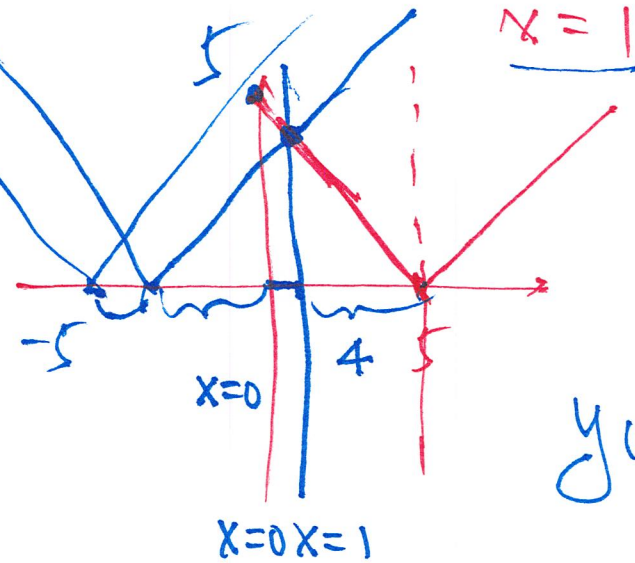
$$y = \sqrt{2^2 - 1^2} = \sqrt{3}$$

#4 (ii) sym. w.r.t. $y = -3$, $y = e^x$

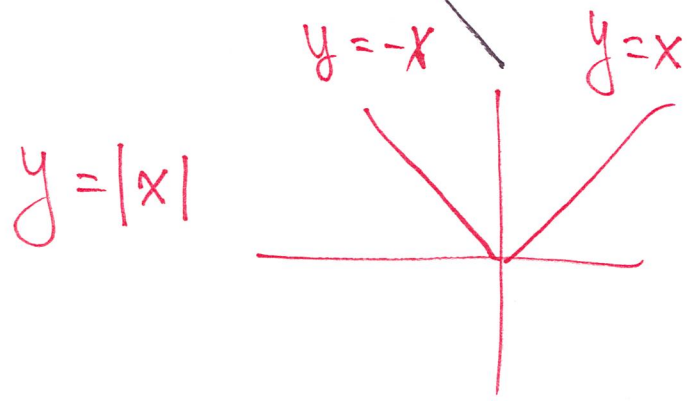


$$y = -6 - e^x$$

$$(ii) y = |x - 5| = \begin{cases} x - 5, & x - 5 > 0 \\ 5 - x, & x - 5 \leq 0 \end{cases}$$



$$y(x) = |(x - 2) + 5|$$



$$y = |-x - 5| = |x + 5|$$

#5 $y = \frac{1 - e^{-x}}{1 + e^{-x}} \Rightarrow$

$y(1 + e^{-x}) = 1 - e^{-x}$

$(y+1)e^{-x} = 1-y \Rightarrow e^{-x} = \frac{1-y}{1+y}$

$-x = \ln \frac{1-y}{1+y} \Rightarrow x = \ln \frac{1+y}{1-y}$

$x = \ln \left(\frac{1+y}{1-y} \right)$

$y = \ln \frac{1+x}{1-x} = \ln(1+x) - \ln(1-x)$

$\frac{1+x}{1-x} > 0$

$1+x > 0$ and $1-x > 0$
 $x > -1$, $x < 1$ $\Rightarrow -1 < x < 1$

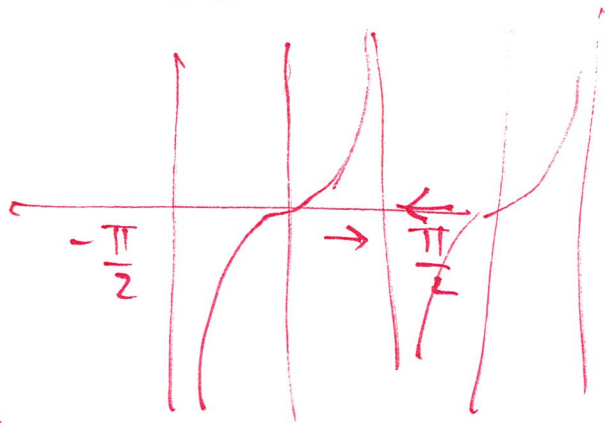
#7

$$(i) \lim_{x \rightarrow 5^-} \frac{x^2 - 3x - 5}{|x-5|} = \infty$$

$25 - 15 - 5 = 5$

$$(ii) \lim_{x \rightarrow \frac{\pi}{2}^-} \tan x = \infty$$

$$\lim_{x \rightarrow \frac{\pi}{2}^+} \tan x = -\infty$$



$$(iii) \lim_{x \rightarrow 0} \left(\frac{5}{x^2 - x} + \frac{5}{x} \right) = \lim_{x \rightarrow 0} \frac{5 + 5(x-1)}{x(x-1)} = 5x = -5$$

$$(iv) \lim_{x \rightarrow \infty} \left(\sqrt{x^2 + 3x + 1} - x \right) \cdot \frac{\sqrt{\quad} + x}{\sqrt{\quad} + x}$$

$$= \frac{4 - 3x}{\sqrt{\quad} + x} = \frac{4 + 5x}{\sqrt{\quad} + x}$$

$\frac{4}{3}$

$$(v) \lim_{x \rightarrow 0} \frac{|3x-4| - |5x+4|}{x} = \frac{|3x-4| + |5x+4|}{|3x-4| + |5x+4|}$$