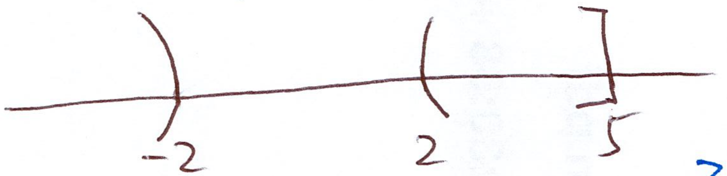
# Study Guide for Final Exam

## #1 Find the domain of f(x)

1.1 $f(t)=\sqrt{5-t}+\dfrac{1}{\sqrt{t^2-4}}$

$$\begin{cases} 5-t\geq 0 \Rightarrow t\leq 5 \\ t^2-4>0 \Rightarrow t^2>4 \Rightarrow |t|>2 \Leftrightarrow t<-2 \text{ and } t>2 \end{cases}$$

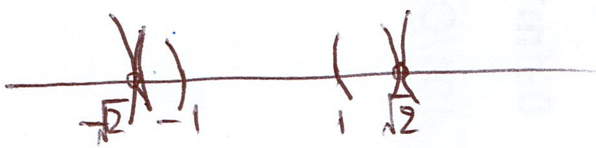


$(-\infty,-2)\cup(2,5]$

1.2 $f(x)=\dfrac{1}{\ln(x^2-1)}$

$$\begin{cases} x^2-1>0 \Rightarrow x^2>1 \Leftrightarrow x>1 \text{ or } x<-1 \\ x^2-1\neq 1 \Rightarrow x^2\neq 2 \Leftrightarrow x\neq\pm\sqrt{2} \end{cases}$$

$(-\infty,-\sqrt{2})\cup(-\sqrt{2},-1)\cup(1,\sqrt{2})\cup(\sqrt{2},\infty)$



1.3 $f(x)=\sqrt{e^{2x}-2e^x+\frac{3}{4}}$

$u=e^x$

$e^{2x}-2e^x+\frac{3}{4}\geq 0$

$\shortparallel$

$u^2-2u+\frac{3}{4}\geq 0$

$(u-1)^2-\frac{1}{4}\geq 0$

$(u-1)^2\geq\frac{1}{4} \Leftrightarrow |u-1|\geq\frac{1}{2}$

$u-1\geq\frac{1}{2}$ or $u-1\leq-\frac{1}{2}$

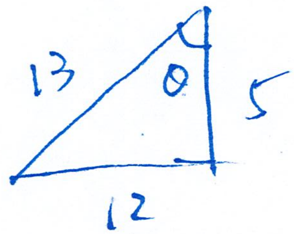
$e^x\geq\frac{3}{2}$ or $e^x\leq\frac{1}{2}$

$x\geq\ln\frac{3}{2}$ or $x\leq\ln\frac{1}{2}$

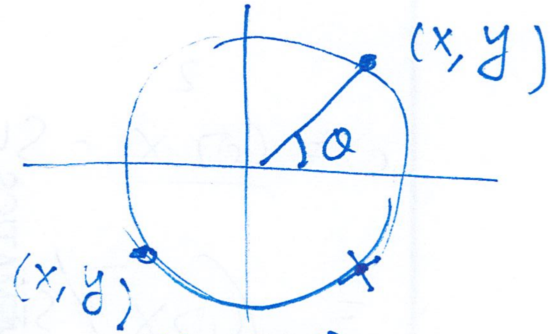
$(-\infty,\ln\frac{1}{2}]\cup[\ln\frac{3}{2},\infty)$

## #2

2.1 $\sin\theta = -\frac{12}{13}$, $\pi < \theta < \frac{3\pi}{2}$, $\cot\theta = \frac{5}{12}$



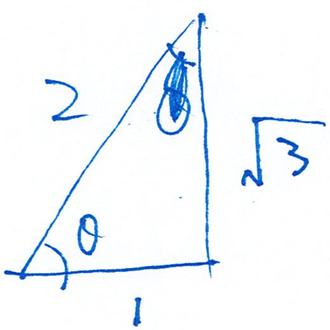

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{5}{12}$$




$x = \cos\theta$

$y = \sin\theta$

2.2 $\sin\theta = -\frac{\sqrt{3}}{2}$, $\frac{3\pi}{2} < \theta < 2\pi$, $\tan\theta = -\sqrt{3}$




$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\tan\theta = \sqrt{3}$$

# #3

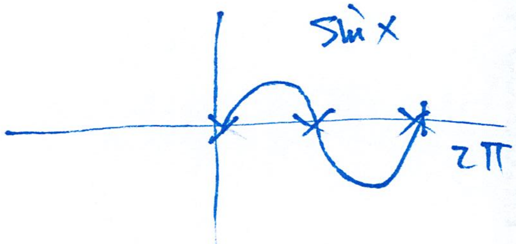
**3.1** $\sqrt{3}\sin x = \sin(2x)$ how many solutions in $[0, 2\pi]$?

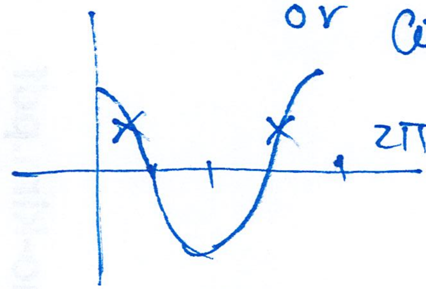
$$\sin(2x) = 2\sin x\cos x$$

$$\sin x\left(\sqrt{3} - 2\cos x\right) = 0 \Longrightarrow \sin x = 0 \Longrightarrow 3$$

$$\text{or } \cos x = \frac{\sqrt{3}}{2} \Longrightarrow 2$$

$$\Longrightarrow 5$$



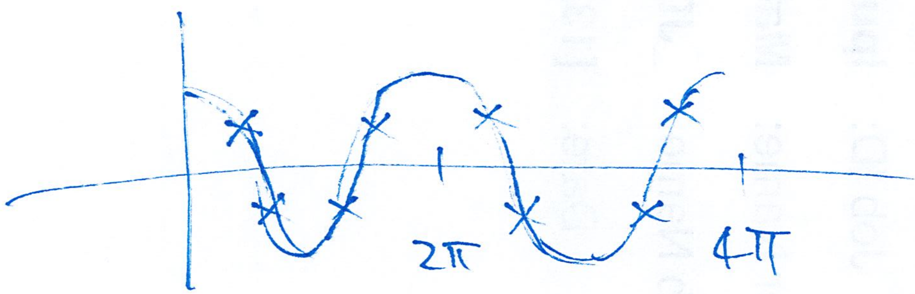

**3.2** Find all $x$ in $[0, 2\pi]$ satisfying $\cos(2x) - \sin(2x) = 0$

$$\cancel{\cos^2 x - \sin^2 x = 2\sin x\cos x}$$

$$\cos(2x) = \sqrt{1 - \cos^2(2x)}$$

$$\cos^2(2x) = 1 - \cos^2(2x)$$

$$\cos^2(2x) = \frac{1}{2} \qquad \cos 2x = \pm\frac{\sqrt{2}}{2}$$




$$\cos(2x) = \pm\frac{\sqrt{2}}{2} \Longrightarrow 2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4}, \cancel{\frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}}$$

$$\Longrightarrow x = \frac{\pi}{8}, \cancel{\frac{3\pi}{8}}, \frac{5\pi}{8}, \cancel{\frac{7\pi}{8}}, \frac{9\pi}{8}, \cancel{\frac{11\pi}{8}}, \frac{13\pi}{8}, \cancel{\frac{15\pi}{8}}$$

$\cos(2x) = \sin(2x) \Longrightarrow x = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$ are not solutions.

Find the interval where $f(x) = 4x^3 - 6x^2 + 3x$ takes the value 2

A. $(-1, 0)$

B. $(0, 1)$

C. $(1, 2)$

D. $(2, 3)$

E. $(3, 4)$

$g(x) = 4x^3 - 6x^2 + 3x - 2$

$f(x) = 2$

$g(-1) = -4 - 6 - 3 - 2$

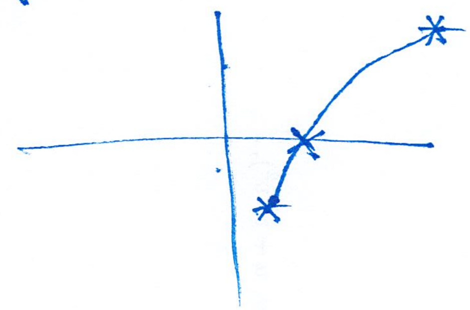
$g(x) = f(x) - 2 = 0$

$g(0) = -2$, $g(1) = 4 - 6 + 3 - 2 = -1$

$g(2) = 4 \cdot 2^3 - 6 \cdot 2^2 + 3 \cdot 2 - 2$

$= 2^3 + 4 > 0$

$(1, 2)$



Find the number of solutions $\boxed{\text{on } [0, 2\pi]}$ of $2\cos x = \sin x + 1$ on $[0, 2\pi]$

A. 0

B. 1

C. 2

D. 3

E. 4

$2\sqrt{1 - \sin^2 x} = \sin x + 1$

$u = \sin x$

$2\sqrt{1 - u^2} = u + 1$

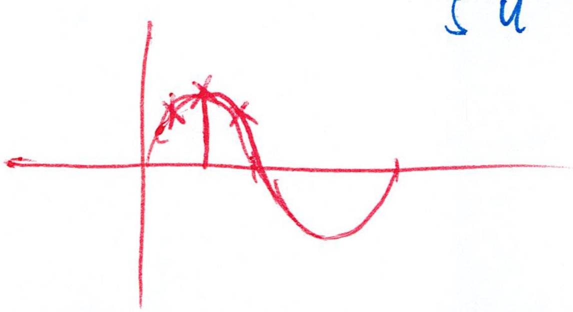
$4(1 - u^2) = (u+1)^2 = u^2 + 2u + 1$

$5u^2 + 2u - 3 = 0 \Rightarrow u = -1$ or $\frac{3}{5}$

i.e., $\sin x = 1$ or $\sin x = \frac{3}{5}$

$\Downarrow$

$x = \frac{\pi}{2}$



#4 Find the inverse of

4.1 $f(x) = \frac{6x-1}{2x+1} = y \Rightarrow 6x-1 = y(2x+1)$

$(6-2y)x = y+1 \Rightarrow x = \frac{y+1}{6-2y}$

$y = \frac{x+1}{6-2x}$

4.2 $f(x) = \frac{2e^x-1}{2e^x+1} = y \Rightarrow 2e^x-1 = y(2e^x+1)$

$2(1-y)e^x = y-1$

$e^x = \frac{y-1}{2(1-y)}$

$y = \ln\frac{x-1}{2(1-x)}$

$x = \ln\frac{y-1}{2(1-y)}$

#5

#18 (WA, HW#5) For what value of $c$, is $f(x)$ cont. on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x, & x < 3 \\ x^3 - cx, & x \geq 3 \end{cases}$$

$$f(3) = 3^3 - 3c$$

$$\lim_{x \to 3^-} f(x) = 9c + 6$$

#5.2 Find $a$ and $b$ so that $f(x) = \begin{cases} x^2 - a, & x \leq 1 \\ \dfrac{3x^2 + 12x - b}{x^2 + 2x - 3}, & x > 1 \end{cases}$ is cont. on $(-\infty, +\infty)$.

$$f(1) = 1 - a$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{3x^2 + 12x - b}{x^2 + 2x - 3}$$

$$3 \cdot 1^2 + 12 \cdot 1 - b = 0$$

$$\Rightarrow b = 15$$

$$= \lim_{x \to 1^+} \frac{3\overbrace{(x^2 + 4x - 5)}^{=(x-1)(x+5)}}{(x-1)(x+3)} = \cancel{18}\frac{3 \cdot 6}{4} = \frac{9}{2}$$

$$1 - a = \cancel{18}\ \tfrac{9}{2} \Rightarrow \cancel{a = -17}$$

$$a = -\frac{7}{2}$$

#6 Given $f'(2) = 5$

6.1 let $g(x) = f(2x)$, $g'(1) = f'(2) \cdot 2 = 5 \cdot 2 = 10$

$(2+4h) - 2 = 4h$

6.2 $\lim_{h \to 0} \frac{f(2+4h) - f(2)}{3h} = \lim_{h \to 0} \left( \frac{f(2+4h) - f(2)}{4h} \right) \frac{4}{3} = \frac{4}{3} f'(2) = \frac{4}{3} \cdot 5 = \frac{20}{3}$

6.3 $\lim_{h \to 0} \frac{f(2+4h) - f(2+5h)}{9h} = -\frac{1}{9} \lim_{h \to 0} \frac{f(2+4h) - f(2+5h)}{-h} = -\frac{1}{9} f'(2) = -\frac{5}{9}$

$(2+4h) - (2+5h)$

$= -h$

#7

7.1 #22 (P149) If the tangent line to $y = f(x)$ at $(4, 3)$ passes through $(0, 2)$.
Find $f(4)$ and $f'(4)$

$y'(4) = f'(4) \Rightarrow y - 3 = f'(4)(x-4)$ tangent line

$f(4) = 3 \qquad 2 - 3 = f'(4)(0-4) \Rightarrow f'(4) = \frac{1}{4}$

#21 (P149) If an equation of the tangent line to the curve $y = f(x)$ at the point where $a = 2$ is $y = 4x - 5$, find $f(2)$ and $f'(2)$.

$$y - f(a) = f'(a)(x - a) \Rightarrow y = \left[f(a) - af'(a)\right] + f'(a)\,x$$

$$= -5 + x$$

$$\Rightarrow \begin{cases} f(a) - af'(a) = -5 \\ f'(a) = 1 \end{cases} \xRightarrow{a=2} \begin{cases} f(2) - 2f'(2) = -5 \\ f'(2) = 1 \end{cases} \Rightarrow f(2) = 2f'(2) - 5$$

$$= 2 - 5 = -3$$

7.2 Find the equation of the line that is tangent to the curve $y = \frac{2}{3} x \sqrt{x}$ and is also parallel to the line $y = 3 + 2x$.

$$= \frac{2}{3} x^{\frac{3}{2}}$$

$$y' = x^{\frac{1}{2}} \Longrightarrow x^{\frac{1}{2}} = 2 \quad (\text{parallel to } y = 3 + 2x) \Longrightarrow x = 4$$

$$\Rightarrow y = \frac{2}{3} 4^{\frac{3}{2}} = \frac{2}{3} \cdot 2^3 = \frac{16}{3}$$

$$y - \frac{16}{3} = 2(x - 4)$$

7.3 Find the equation(s) of the tangent line(s) to the graph of a function $y = x^2$, passing $(1, -3)$.

$$y' = 2x, \quad \boxed{\cancel{y'(1) = 2}} \Longrightarrow \text{the tangent lines passing } (1, -3)$$

$$y + 3 = m(x - 1)$$

$$\Rightarrow 2x = m \Rightarrow x = \frac{m}{2}$$

$$y = \left(\frac{m}{2}\right)^2$$

$$\cancel{\text{intersection of } y = x^2 \text{ and } y = -3 + m(x-1)}$$

$$\cancel{\Rightarrow x^2 = -3 + mx - m \Rightarrow x^2 - mx + (3 + m) = 0}$$

$$\cancel{\Rightarrow x = \frac{m \pm \sqrt{m^2 - 4(3+m)}}{2}}$$

$$\xrightarrow{y = -3 + m(x-1)} \left(\frac{m}{2}\right)^2 = m \cdot \frac{m}{2} - 3 - m$$

$$= \frac{1}{2} m^2 - 3 - m$$

$$\Longrightarrow 0 = m^2 - 4m - 12 = (m - 6)(m + 2)$$

$$\Longrightarrow m = -2 \text{ or } m = 6 \Longrightarrow y = -3 + 6(x - 1)$$

$$\text{or } y = -3 - 2(x - 1)$$

#8 compute derivatives

8.1 $y = \sin(\sin(\sin x))$, $y' = \cos(\sin(\sin x)) \cdot \cos(\sin x) \cdot \cos x$

8.2 $y = \left(\frac{t-2}{2t+1}\right)^9$, $y' = 9\left(\frac{t-2}{2t+1}\right)^8 \cdot \frac{(2t+1) - 2(t-2)}{(2t+1)^2} = 9\left(\frac{t-2}{2t+1}\right)^8 \frac{5}{(2t+1)^2}$

8.3 $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$, $y' = \frac{1}{2} \cdot \left(x + \sqrt{x + \sqrt{x}}\right)^{-\frac{1}{2}} \left[1 + \frac{1}{2}\left(x + \sqrt{x}\right)^{-\frac{1}{2}} \left(1 + \frac{1}{2} x^{-\frac{1}{2}}\right)\right]$

8.4 $y = e^{\sec 3\theta}$, $y' = e^{\sec(3\theta)} \cdot \sec(3\theta)\tan(3\theta) \cdot 3$

8.5 $y = e^{2^{x^3}}$, $y' = e^{2^{x^3}} \cdot \left(2^{x^3}\right)' = e^{2^{x^3}} \cdot 2^{x^3} \cdot \ln 2 \cdot 3x^2$

8.6 $F(x) = f(x)^2 \cdot f(g(x))$ and

$$\begin{cases} f(1)=5, & f(2)=3, & f(3)=-1 \\ f'(1)=4, & f'(2)=3, & f'(3)=-2 \\ g(1)=3, & g(2)=2, & g(3)=-1 \\ g'(1)=2, & g'(2)=3, & g'(3)=4 \end{cases}$$

$$F'(x) = 2f(x)\cdot f'(x)\cdot f(g(x)) + f^2(x)\, f'(g(x))\, g'(x)$$

$$F'(1) = 2f(1)f'(1)f(g(1)) + f^2(1)\, f'(g(1))\, g'(1)$$

$$= 2\cdot 5\cdot 4\cdot f(3) + 5^2 f'(3)\cdot 2$$

$$= 40\cdot(-1) + 50\cdot(-2) = -40-100 = -140$$

9. derivative of $y = f(x)^{g(x)} = e^{g(x)\ln f(x)}$, $y' = f(x)^{g(x)} \cdot \left[g'(x)\ln f(x) + \frac{g(x) f'(x)}{f(x)}\right]$

$\underline{\ln y = g(x)\ln f(x)} \rightarrow \frac{1}{y} \cdot y' =$

9.1 $y = x^x = e^{x\ln x}$

$y' = e^{x\ln x}\left[\ln x + x\frac{1}{x}\right] = x^x\left[\ln x + 1\right]$

9.2 $y = (\ln x)^{\tan 3x} = e^{\underline{\tan(3x)\ln(\ln x)}}$

$y' = (\ln x)^{\tan 3x} \cdot \left[\sec^2(3x) \cdot 3\ln(\ln x) + \tan(3x)\frac{1}{\ln x} \cdot \frac{1}{x}\right]$

9.3 $y = (\sqrt{x})^{\sin x} = \left(x^{\frac{1}{2}}\right)^{\sin x} = x^{\frac{1}{2}\sin x} = e^{\frac{1}{2}\sin x\ln x}$

$y' = (\sqrt{x})^{\sin x}\frac{1}{2}\left[\cos x\ln x + \sin x\frac{1}{x}\right]$

## #10 implicit differentiation

10.1 $\begin{cases} \underline{0 = f(x) + x^2 (f(x))^3} \\ f(1) = 2 \end{cases}$ find $f'(1)$

$$0 = \underline{f'(x)} + 2x f^3(x) + \underline{x^2 \cdot 3 f^2(x) \cdot f'(x)}$$

$$= (1 + 3x^2 f^2(x)) f'(x) + 2x f^3(x)$$

$$f'(x) = -\frac{2x f^3(x)}{1 + 3x^2 f^2(x)}$$

$$f'(1) = -\frac{2 f^3(1)}{1 + 3 f^2(1)} = -\frac{2 \cdot 2^3}{1 + 3 \cdot 2^2}$$

10.2 find the slope of $y(x)$ given by $\underline{x^2 + 2xy - y^2 + x = 0}$ at $(1, 2) = (x, y)$

$$0 = 2x + [2y + 2x\underline{y'}] - \underline{2y \cdot y'} + 1$$

$$= (2x + 1 + 2y) + 2(x - y)y' \Rightarrow$$

$$y' = \frac{2x + 1 + 2y}{2(y - x)}$$

$$y'(1) = \frac{2 \cdot 1 + 1 + 2 \cdot 2}{2(2 - 1)}$$

10.3 given $e^{\frac{x}{y}} = 7x - y$, find $\frac{dy}{dx}$

$$e^{\frac{x}{y}} \cdot \frac{y - xy'}{y^2} = 7 - y'$$

#11 linear approximation $f(x) \approx f(a) + f'(a)(x-a)$ at $a = x$

11.1 $f(x) = e^x$, find its linear approx at $a=0$. Estimate $e^{0.01}$

$f(0) = 1$

$f'(x) = e^x$

$f'(0) = 1$

$$e^x \approx 1 + 1\cdot(x-0) = 1+x$$

$$e^{0.01} \approx 1 + 0.01 = 1.01$$

11.2 Estimate $\sqrt[3]{26.8}$ using a linear approx. $f(x) = ?$ $a = ?$

$f(x) = x^{\frac{1}{3}}$ $a = 27$

$$f(x) = x^{\frac{1}{3}} \approx f(27) + f'(27)(x-27) = 3 + \frac{1}{27}(x-27)$$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$f(27) = 27^{\frac{1}{3}} = (3^3)^{\frac{1}{3}} = 3$

$f'(27) = \frac{1}{3}\cdot 3^{-2} = \frac{1}{27}$

$$f(26.8) \approx 3 + \frac{1}{27}(26.8-27)$$

$$= 3 + \frac{-0.2}{27}$$

#12 moving particle

#8 (Webassign HW 14) If a ball is thrown vertically upward with a velocity 144 ft/s then its height after $t$ seconds is $s(t)=144t-16t^2$. (a) what is the max. height? (b) what is the velocity of the ball when it is 320 ft above the ground ~~when~~ on its way up? and on its way down?

(a) $v(t)=0$ $\cancel{s'(t)}$ $\quad v(t)=s'(t)=144-32t$

$0=144-32t \Rightarrow t=\frac{144}{32}=4.5 \Rightarrow s(4.5)=$

(b) $320=s(t)=\cancel{144t-32}\ 144t-16t^2 \Rightarrow 0=t^2-9t+20$

$=(t-4)(t-5)$

$\Rightarrow t=4$, $t=5$ $\quad \cancel{s(4)}.\ v(4)=144-32\cdot 4$

$v(5)=$ - -

12.2 $s(t)=t^3-6t^2+9t$, find the total distance during the first 6 seconds.

$v=3t^2-12t+9$

$=3(t^2-4t+3)$

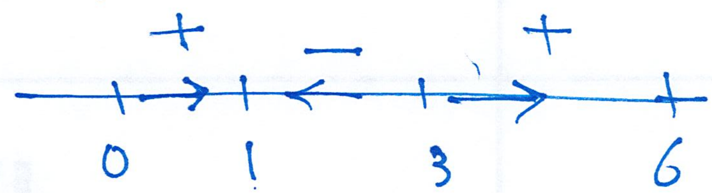
$=3(t-1)(t-3)$

$$d=\int_0^6 |v(t)|\,dt = 3\int_0^6 |t-1||t-3|\,dt$$

$s=|\vec{v}(t)|$

$$=3\left[\int_0^1 (t^2-4t+3)\,dt+\int_1^3(-t^2+4t-3)\,dt\right.$$

$$\left.+\int_3^6 (t^2-4t+3)\,dt\right]$$




12.3

#13

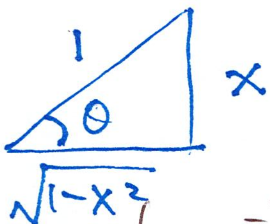
#13.1 $\tan\left(2\sin^{-1}x\right) = \dfrac{\sin 2\theta}{\cos 2\theta} = \dfrac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \dfrac{2\cdot x\cdot\sqrt{1-x^2}}{(1-x^2)-x^2}$

$\theta = \sin^{-1}x \iff \sin\theta = x$

$\sin^2\theta + \cos^2\theta = 1$

$\sin(A+B) = \sin A\cos B + \sin B\cos A$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$




$\cos\theta = \sqrt{1-x^2}$

13.2 $\cos\left(2\sin^{-1}x\right)$

$\theta = \sin^{-1}x \iff \sin\theta = x$

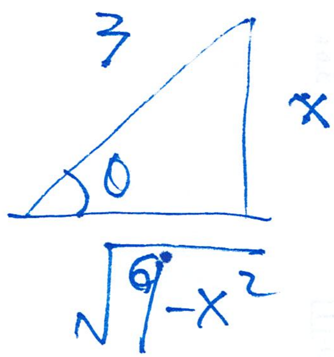
$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$= \left(\sqrt{1-x^2}\right)^2 - x^2 = 1-2x^2$

13.3 $\cos\left(\tan^{-1}\dfrac{x}{\sqrt{9-x^2}}\right)$

$\tan^{-1}\dfrac{x}{\sqrt{9-x^2}} = \theta$

$\tan\theta = \dfrac{x}{\sqrt{9-x^2}}$




$\cos\theta = \frac{1}{3}\sqrt{9-x^2}$

# #14 hyperbolic functions

**14.1** $\sinh(0) = \frac{1}{2}\left(e^{0} - e^{-0}\right) = 0$

$$\sinh(\ln 5) = \frac{1}{2}\left(e^{\ln 5} - e^{-\ln 5}\right) = \frac{1}{2}\left(5 - \frac{1}{5}\right)$$

$$\frac{1+\tanh\left(\frac{1}{2}\right)}{1-\tanh\left(\frac{1}{2}\right)} = \frac{1+\frac{\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)}}{1-\frac{\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)}} = \frac{\cosh\left(\frac{1}{2}\right)+\sinh\left(\frac{1}{2}\right)}{\cosh\left(\frac{1}{2}\right)-\sinh\left(\frac{1}{2}\right)} = \frac{\frac{1}{2}\left(e^{\frac{1}{2}}+e^{-\frac{1}{2}}\right)+\frac{1}{2}\left(e^{\frac{1}{2}}-e^{-\frac{1}{2}}\right)}{\frac{1}{2}\left(e^{\frac{1}{2}}+e^{-\frac{1}{2}}\right)-\frac{1}{2}\left(e^{\frac{1}{2}}-e^{-\frac{1}{2}}\right)}$$

$\cancel{\sinh\left(\frac{1}{2}\right)}$

$$= \frac{e^{\frac{1}{2}}}{e^{-\frac{1}{2}}} = e$$

**14.2** $f(x) = \sinh(\ln x)$,

$$f'(x) = \cosh(\ln x)\cdot\frac{1}{x}$$

$$f'(5) = \cosh(\ln 5)\cdot\frac{1}{5} = \frac{1}{2}\left(e^{\ln 5} + e^{-\ln 5}\right)\cdot\frac{1}{5} = \frac{1}{10}\left(5 + \frac{1}{5}\right)$$

# #15 Two "Related Rates"

- snowball problem (Lesson 16, #14) (Study Guide 2)
- light house problem (Exam 2 Version 1, #10) (Study Guide 2)
- Inverted circular conical tank problem (Grave problem) ← Study Guide 2
  (Lesson 16, #25) (Exam 2 Version 1, #12)
  (Lesson 15, Ex. 3)
- kite problem (Study Guide 2)
- man walking along a straight path with a search light chasing him
  (study Guide 2)
- two cars (ships) moving east-west, south-north problem (Study Guide 2)

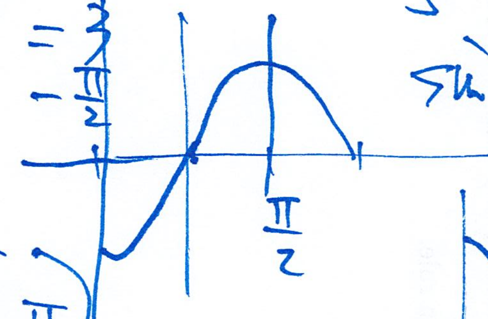
#16

$\tan\left(\sin^{-1}\frac{4}{5}\right) = \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{4}{3}$

$\theta = \sin^{-1}\frac{4}{5} \iff \sin\theta = \frac{4}{5}$


$\sqrt{5^2-4^2} = 3$

$\cos\theta = \frac{3}{5}$




$\sin\theta$

$\sin^{-1}\left(\sin\left(\frac{7\pi}{3}\right)\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right) = \frac{\pi}{3}$

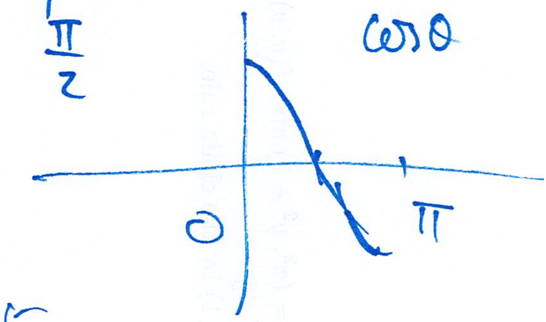
$\sin^{-1}x$ $\quad [0,\pi]$ $\quad \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\frac{7\pi}{3} = \frac{6\pi+\pi}{3} = 2\pi + \frac{1}{3}\pi$

$\cos^{-1}x$, $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $\quad [0,\pi]$

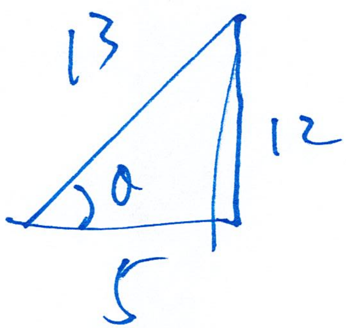
$\sin\frac{7\pi}{3} = \sin\left(2\pi + \frac{1}{3}\pi\right) = \sin\frac{1}{3}\pi$

$\cos\theta$




$\sin\left(2\sin^{-1}\left(\frac{12}{13}\right)\right) = \sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{12}{13}\cdot\frac{5}{13}$

$\sin\theta = \frac{12}{13}$

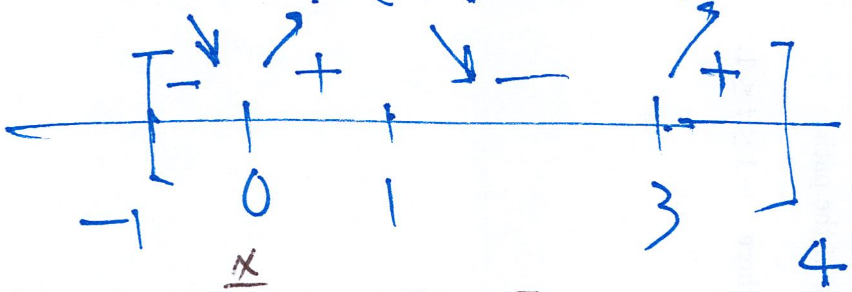


# #17

17.1 Find the absolute max/min and local max/min of $f(x)=3x^4-16x^3+18x^2$ on $[-1,4]$

$$f'(x) = 12x^3-3\cdot 16x^2+2\cdot 18x = 12x\left[x^2-4x+3\right]$$

$$= 12x(x-1)(x-3)$$




l. min $f(0) = 0$

l. max $f(1) = 3-16+18 = 5$

abs. min $f(3) = 3\cdot 3^4-16\cdot 3^3+18\cdot 3^2 = -3^3 = -27$

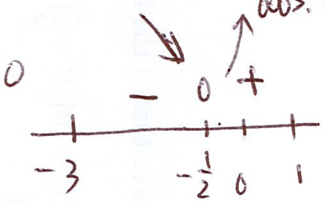
abs. max $f(-1) = 3+16+18 = 37$

$f(4) = 3\cdot 4^4-16\cdot 4^3+18\cdot 4^2 = 2\cdot 4^2 = 32$

• $f(x) = xe^{\frac{x}{2}}$ on $[-3,1]$

$$f'(x) = e^{\frac{x}{2}}+\tfrac{1}{2}xe^{\frac{x}{2}} = \left(1+\tfrac{1}{2}x\right)e^{\frac{x}{2}} = 0$$

$\Longrightarrow x = -\frac{1}{2}$




$f(-3) = -3e^{-\frac{3}{2}} = -\dfrac{3}{e^{\frac{3}{2}}}$ abs. min

$f(-\frac{1}{2}) = -\dfrac{1}{2e^{\frac{1}{2}}}$ l. min

$f(1) = e^{\frac{1}{2}}$ abs. max

- $f(x) = (x-1)^3$ on $[-1, 3]$, $f'(x) = 3(x-1)^2 = 0 \Rightarrow x = 1$


$f(-1) = (-1)^3 = -8$ abs. min

$f(0) = 1$

$f(3) = 2^3 = 8$ abs. max.

- $f(x) = 2\cos x + \sin 2x$ on $[0, \frac{\pi}{2}]$, $f'(x) = -2\sin x + 2\cos 2x = -2\sin x + 2(\cos^2 x - \sin^2 x)$

$= -2\sin x + 2(1 - 2\sin^2 x) = -4\left(\sin^2 x + \frac{1}{2}\sin x - \frac{1}{2}\right.$

$= -2(2\sin^2 x + \sin x - 1) = -4\left(\sin^2 x + \frac{1}{2}\sin x - \frac{1}{2}\right)$

$= -4(\sin x + 1)(\sin x - \frac{1}{2})$

$\Rightarrow \sin x = -1$ or $\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$.

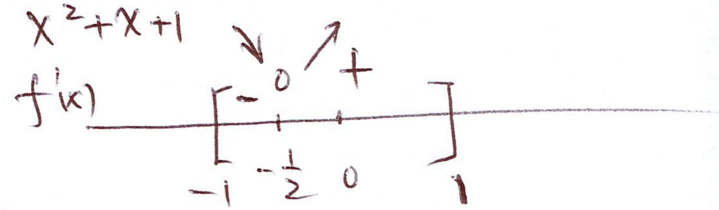
$f(0) = 2$

$f(\frac{\pi}{6}) = 2 \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \frac{3}{2}\sqrt{3}$ abs. max

$f(\frac{\pi}{2}) = 0$ abs. min

- $f(x) = \ln(x^2 + x + 1)$ on $[-1, 1]$, $f'(x) = \frac{2x+1}{x^2+x+1} = 0 \Rightarrow x = -\frac{1}{2}$

$x^2 + x + 1 = (x + \frac{1}{2})^2 + \frac{3}{4} >$



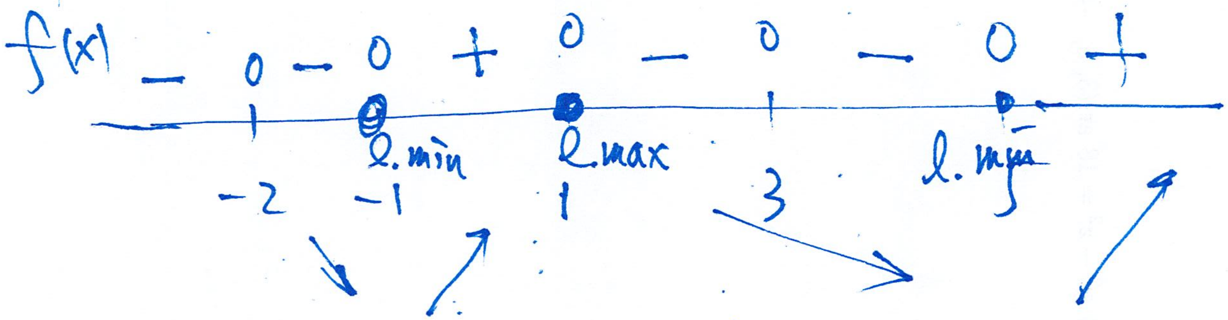

$f(-1) = \ln 1 = 0$

$f(-\frac{1}{2}) = \ln\left(\frac{1}{4} - \frac{1}{2} + 1\right) = \ln\frac{3}{4} = \ln 3 - \ln 4 < 0$ abs. min

$f(1) = \ln 3$ abs. max

# #18

**18.1** $f'(x)=\cancel{(x+2)^2}(x+1)(x-1)^3\cancel{(x-3)^2}(x-5)$. Find $x$ s.t. $f$ has local max./min.

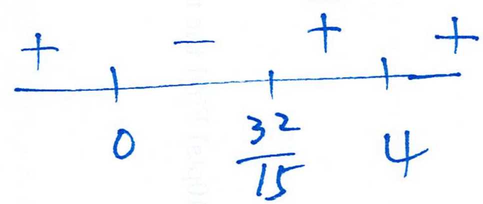



**18.2** $f(x)=x^8(x-4)^7$

(a) find critical numbers, (b) $2^{nd}$-der. test, (c) $1^{st}$-der. test.

(a)
$$0=f'(x)=8x^7(x-4)^7+7x^8(x-4)^6=x^7(x-4)^6\left[8x-32+7x\right]=x^7(x-4)^6(15x-32)$$

$$\Longrightarrow x=0,\ x=4,\ x=\frac{32}{15}$$




(c) $f(0)$ l. max

$f\left(\frac{32}{15}\right)$ l. min

$f(4)$ neither l. max or l. min

(b)
$$f''(x)=7x^6(x-4)^6(15x-32)+6x^7(x-4)^5(15x-32)+15x^7(x-4)^6$$

$$f''(0)=0,\quad f''\left(\tfrac{32}{15}\right)=15\cdot\left(\tfrac{32}{15}\right)^7\left(\tfrac{32}{15}-4\right)^6>0$$

$$f''(4)=0$$

$\Rightarrow f\left(\frac{32}{15}\right)$ — local min ~~abs min~~

$f(0)$, $f(4)$ undetermined

#19 $\frac{0}{0}$, $\frac{\infty}{\infty}$

$$\lim_{x\to\infty}\frac{\ln x}{\sqrt{x}} = 0 = \lim_{x\to\infty}\frac{\frac{1}{x}}{\frac{1}{2}x^{-\frac{1}{2}}} = 2\lim_{x\to\infty}\frac{x^{\frac{1}{2}}}{x} = 2\lim_{x\to\infty}\frac{1}{x^{1/2}} = 0$$

$$\lim_{x\to 0}\frac{1-\cos x}{3x^2} = \lim_{x\to 0}\frac{\sin x}{6x} = \frac{1}{6} = \lim_{x\to 0}\frac{\cos x}{6} = \frac{1}{6}$$

$$\lim_{x\to 0}\frac{\sin x}{1-x^2} = 0$$

$$\lim_{x\to 0}\frac{\cos x}{-2x} \neq \lim_{x\to 0}\frac{-\sin x}{-2} = 0$$

$$\lim_{x\to 0}\frac{7^x-6^x}{3^x-2^x} = \lim_{x\to 0}\frac{7^x\ln 7 - 6^x\ln 6}{3^x\ln 3 - 2^x\ln 2} = \frac{\ln 7-\ln 6}{\ln 3-\ln 2}$$

#20 $\pm\infty \times 0$, $\infty - \infty$

$$\boxed{\lim_{x\to 0}\frac{\sin x}{x} = 1}$$

$$\lim_{x\to 0^+} \underbrace{\sin(x)}_{\downarrow\, 0}\, \underbrace{\ln(2x)}_{\to -\infty} = \lim_{x\to 0^+} \frac{\ln(2x) \to \infty}{\frac{1}{\sin x} \nearrow \infty} = \lim_{x\to 0^+} \frac{\frac{1}{\cancel{2}x}\cdot \cancel{2}}{(-1)\sin^{-2} x \cdot \cos x} = -\lim_{x\to 0^+} \frac{\sin^2 x}{x\cos x} = -\lim_{x\to 0^+} \underbrace{\frac{\sin x}{x}}_{\to 1} \cdot \underbrace{\frac{\sin x}{\cos x}}_{\to 0}$$

$$= -1 \cdot 0 = 0$$

$$\lim_{x\to\infty} \underbrace{2x}_{\downarrow\,\infty} \underbrace{\tan\left(\frac{1}{3x}\right)}_{\downarrow\, 0} = \lim_{x\to\infty} 2\cdot \frac{\tan\left(\frac{1}{3x}\right)}{\frac{1}{x}} = 2\lim_{x\to\infty} \frac{\sec^2\left(\frac{1}{x}\right)\cdot \cancel{(-1)}(3\cancel{x})^{-2}\cdot 3}{\cancel{(-1)}\cancel{(x)^{-2}}}$$

$$= \frac{2}{3}\lim_{x\to\infty} \frac{1}{\cos^2\left(\frac{1}{x}\right)} = \frac{2}{3}$$

$$\left(\frac{1}{3x}\right)' = \left((3x)^{-1}\right)'$$

$$= -1\,(3x)^{-2}\cdot 3$$

$$\lim_{x\to\left(\frac{\pi}{2}\right)^-} (2x - \pi)\cdot \tan x =$$

$$\lim_{x\to 1} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right) =$$

$$\lim_{x\to 4} \left(\frac{1}{\sqrt{x}-2} - \frac{4}{x-4}\right) =$$

#21 $\lim_{x \to a} [f(x)]^{g(x)}$, $0^0$, $\infty^0$, $1^\infty$

$$\lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^{7x} =$$

$$\lim_{x \to \infty} \left(\frac{2x+1}{2x-1}\right)^{4x+1} =$$

$$\lim_{x \to \infty} \left(2x + e^{5x}\right)^{\frac{1}{x}} =$$

$$\lim_{x \to 0^+} \tan(5x)^{\sin x} =$$

# #23 ONE optimization problem

(1) maximize the area of a rectangle inscribed in a circle (ellipse) (see Lesson 26, #26)

(2) minimize the material used to make a rectangular box (or a circular cylindar)

(Lesson 26, #14) (Lesson 25, Ex. 2)

(3) minimize the time to reach another point across the river by first rowing and then running.

(Lesson 25, Ex. 4)

#21 (P337) Find the point on the line $y = 2x+3$ that is closest to the origin.

$$d(x) = \sqrt{x^2 + y^2} = \sqrt{x^2 + (2x+3)^2}$$

$$f(x) = d^2(x) = x^2 + (2x+3)^2$$

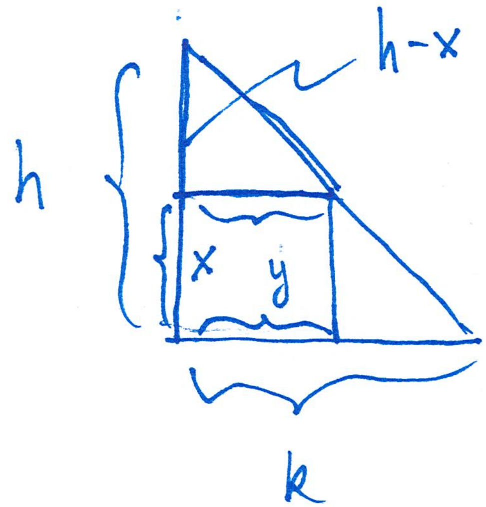
$$f'(x) = 2x + 2(2x+3) \cdot 2 = 2x + 8x + 12 = 10x + 12 = 0$$

$$\Rightarrow x = -\frac{6}{5}$$

$$y = 2 \cdot \left(-\frac{6}{5}\right) + 3 = \frac{-12+15}{5} = +\frac{3}{5}$$

$$\left(-\frac{6}{5}, +\frac{3}{5}\right)$$

Find the dimensions of the rectangle of largest area that can be incribed in a right triangle.




$$A(x) = xy = x\,\frac{k}{h}(h-x)$$

$$A'(x) = \frac{k}{h}\left[(h-x) - x\right] = \frac{k}{h}(h-2x) = 0$$

$$\Rightarrow x = \frac{h}{2} \Rightarrow y = \frac{k}{h}\left(h - \frac{h}{2}\right) = \frac{k}{2}$$

$$A''(x) = -2\,\frac{k}{h} < 0$$

$$\frac{h-x}{h} = \frac{y}{k} \Rightarrow y = \frac{k}{h}(h-x)$$

$$\boxed{\begin{aligned} P(t) &= P(0)\, e^{kt} \\ m(t) &= m(0)\, e^{kt} \end{aligned}}$$

$k > 0$

$k < 0$

$$\boxed{m(h) = \frac{1}{2} \cancel{m(0)}}$$

$\parallel$

$\cancel{m(0)}\, e^{kh}$

$$e^{kh} = \frac{1}{2}$$

$$kh = \ln \frac{1}{2} = -\ln 2$$

$$k = -\frac{\ln 2}{h}$$

$$m(t) = m(0)\, e^{-\frac{t}{h} \ln 2} = m(0) \left(e^{\ln 2}\right)^{-\frac{t}{h}} = m(0)\, 2^{-\frac{t}{h}}$$

#24 the population growth and radioactive decay.

24.1 $\boxed{P(t) = P(0)e^{kt}}$, $P(3) = 3P(0)$

$$3\cancel{P(0)} = P(3) = \cancel{P(0)}\, e^{k\cdot 3} \Rightarrow e^{3k} = 3 \Rightarrow 3k = \ln 3$$

$$k = \frac{1}{3}\ln 3$$

24.2 $P(0) = 50$, $P(3) = 100$, Find $t$ such that $P(t) = 700$

$$100 = P(3) = P(0)e^{k\cdot 3} = 50e^{3k} \Rightarrow e^{3k} = 2 \Rightarrow 3k = \ln 2$$

$$\Rightarrow k = \frac{1}{3}\ln 2$$

$$700 = P(0)e^{\frac{1}{3}\ln 2\cdot t} = 50\, e^{\frac{1}{3}\ln 2\, t}$$

$$e^{\frac{1}{3}\ln 2\cdot t} = \frac{70}{5} = 14 \Rightarrow \frac{t}{3}\ln 2 = \ln 14 \Rightarrow t = \frac{3\ln 14}{\ln 2}$$

24.3 $\quad m(t) = \underline{m(0)}\, 2^{-t/h}$

Given $h = 30$, $m(0) = 60$, Unknown $m(t) = 1$

$$1 = m(t) = m(0)\, 2^{-\frac{t}{h}} = 60 \cdot 2^{-\frac{t}{30}}$$

$$\Rightarrow 2^{-\frac{t}{30}} = \frac{1}{60} \Longrightarrow -\frac{t}{30} = \log_2 \frac{1}{60} = -\log_2 60$$

$$\Rightarrow t = 30 \log_2 60$$

24.4 $\quad$ Given $h = 5730$, $\underline{m(t) = \cancel{m(0)} \cdot 0.74}$

$$\| $$

$$\cancel{m(0)}\, 2^{-\frac{t}{5730}}$$

$$-\frac{t}{5730} = \log_2 0.74 \;\# \Longrightarrow t = -5730 \cdot \log_2 0.74$$

$$\int_{-2}^{0} x|x+1|\,dx = \int_{-2}^{-1} -x(x+1)\,dx + \int_{-1}^{0} x(x+1)\,dx$$

A. 2
B. 1
C. 0
D. -1
E. -2

$$|x+1| = \begin{cases} x+1, & x+1>0 \iff x>-1 \\ -(x+1), & x+1<0 \iff x<-1 \end{cases}$$

$$\lim_{x\to 0} \frac{x^2-x}{|x|} = \begin{cases} \lim_{x\to 0^+} \frac{x^2-x}{x} = \lim_{x\to 0^+}(x-1) = -1 \\ \lim_{x\to 0^-} \frac{x^2-x}{-x} = \lim_{x\to 0^-}(1-x) = 1 \end{cases} \quad \#$$

A. 1
B. -1
C. 0
D. -2
E. DNE.

$$|x| = \begin{cases} x & x>0 \\ -x & x<0 \end{cases}$$

# #25 (the substitution rule)

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx \xrightarrow[du = -\sin x \, dx]{u = \cos x} \int \frac{1}{u}(-du) = -\ln u + C$$
$$= -\ln(\cos x) + C$$

$$\int \frac{\ln x}{x} dx \xrightarrow[du = \frac{1}{x} dx]{u = \ln x} \int u \, du = \frac{1}{2}(\ln x)^2 + C$$

$$\int_0^4 \sqrt{1+2x} \, dx \xrightarrow[du = 2dx]{u = 1+2x : 1 \to 9} \int_1^9 u^{\frac{1}{2}} \frac{1}{2} du = \frac{1}{2} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_1^9 = \frac{1}{3}(3^3 - 1)$$

$$\int_0^2 x^5 \sqrt{1+x^2} \, dx \xrightarrow[du = 2x dx]{u = 1+x^2 : 1 \to 5} \int_1^5 u^{\frac{1}{2}} x^4 \cdot \frac{1}{2} du = \frac{1}{2} \int_1^5 u^{\frac{1}{2}} (u-1)^2 du$$

$x^2 = u - 1$

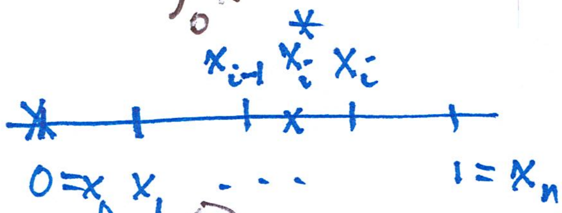
$$\sec^2 x = \frac{1}{\cos^2 x}, \quad 1 + \tan^2 x = 1 + \frac{\sin^2 x}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$\int_0^{\frac{\pi}{4}} \sec^4 x \tan x \, dx \xrightarrow[du = \sec^2 x \, dx]{u = \tan x : 0 \to 1} \int_0^1 \sec^2 x \, u \, du = \int_0^1 (1+u^2) u \, du$$

$$\int_0^{\frac{1}{2}} \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx \xrightarrow[du = \frac{1}{\sqrt{1-x^2}} dx]{u = \sin^{-1} x : 0 \to \frac{\pi}{6}} \int_0^{\frac{\pi}{6}} u \, du = \frac{1}{2} u^2 \Big|_0^{\frac{\pi}{6}} = \frac{1}{2}\left(\frac{\pi}{6}\right)^2$$

#26

**26.1** $\int_0^1 \sqrt{1-x^2}\,dx$ as the Riemann sum dividing $[0,1]$ into $n$ equal subintervals and using the left end points.




$\Delta x = \frac{1}{n}$, $x_i = x_0 + i\cdot\frac{1}{n} = \frac{i}{n}$ $\quad f(x) = \sqrt{1-x^2}$

$$\int_0^1 \sqrt{1-x^2}\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\,\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{1-\left(\frac{i-1}{n}\right)^2}\cdot\frac{1}{n}$$

**26.2** Compute

(i) $\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{3+i\cdot\frac{5}{n}}\cdot\frac{5}{n}$

$\Delta x = \frac{5}{n}$, $x_i = 3+i\frac{5}{n} \Rightarrow x_0 = 3$, $x_n = 3+n\cdot\frac{5}{n} = 3+5 = 8$

$f(x) = \sqrt{x}$

$$= \int_3^8 \sqrt{x}\,dx = \frac{2}{3}x^{\frac{3}{2}}\Big|_3^8 = \frac{2}{3}\left[8^{\frac{3}{2}} - 3^{\frac{3}{2}}\right]$$

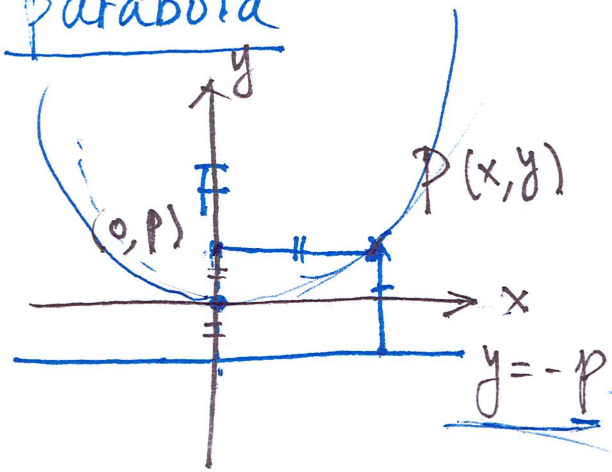
(ii) $\lim_{n\to\infty}\sum_{i=1}^{n}\left(1+i\frac{2}{3n}\right)^5\cdot\frac{2}{3n}$

$\Delta x = \frac{2}{3n}$, $x_i = 1+i\cdot\frac{2}{3n}$ $\begin{cases} x_0 = 1 \\ x_n = 1+n\cdot\frac{2}{3n} = 1+\frac{2}{3} = \frac{5}{3}\end{cases}$

$f(x) = x^5$

$$= \int_1^{\frac{5}{3}} x^5\,dx = \frac{1}{6}x^6\Big|_1^{\frac{5}{3}} = \frac{1}{6}\left[\left(\frac{5}{3}\right)^6 - 1\right]$$

## parabola



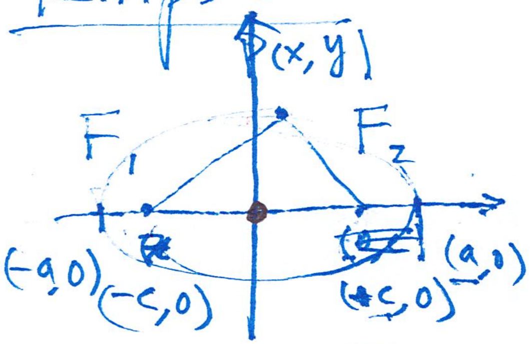

$$|PF| = |y+p|$$

$$\boxed{x^2 = 4py}$$

focus $(0,p)$

$y=-p$ directrix

## Ellipse

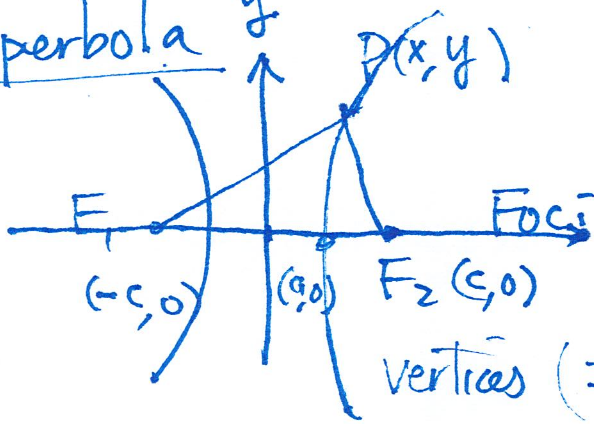



$$|PF_1| + |PF_2| = 2a = \text{constant}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad b^2 = a^2 - c^2$$

$$\begin{cases} \frac{x}{a} + \frac{y}{b} = 0 \\ \frac{x}{a} - \frac{y}{b} = 0 \end{cases} \qquad \left(\frac{x}{a} + \frac{y}{b}\right)\left(\frac{x}{a} - \frac{y}{b}\right) = 1$$

## Hyperbola




$$|PF_1| - |PF_2| = \pm 2a = \text{const}$$

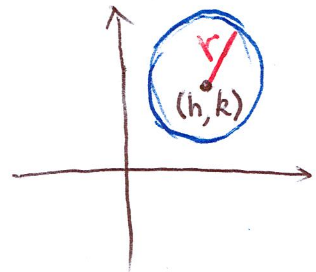
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\cancel{b^2 = a}$$

$$c^2 = a^2 + b^2$$

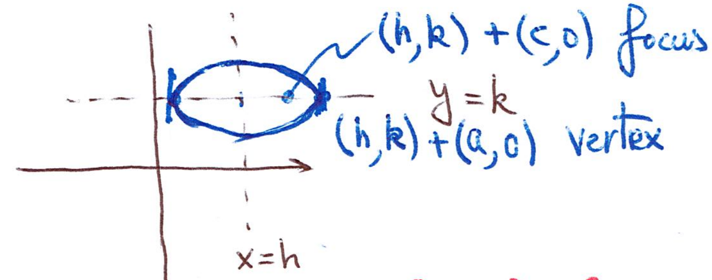
# Shifted Conics

**circle** $(x-h)^2 + (y-k)^2 = r^2$




circle

**ellipse** $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$




$\underline{a > b}$

ellipse

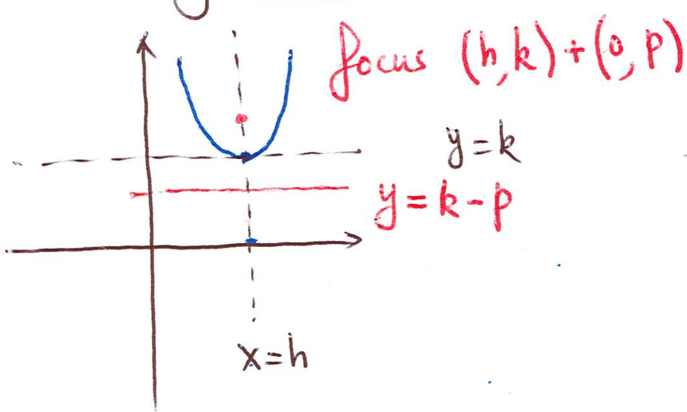
$b^2 = a^2 - c^2$

**parabola** $y - k = a(x-h)^2$ $\quad a = \dfrac{1}{4p}$

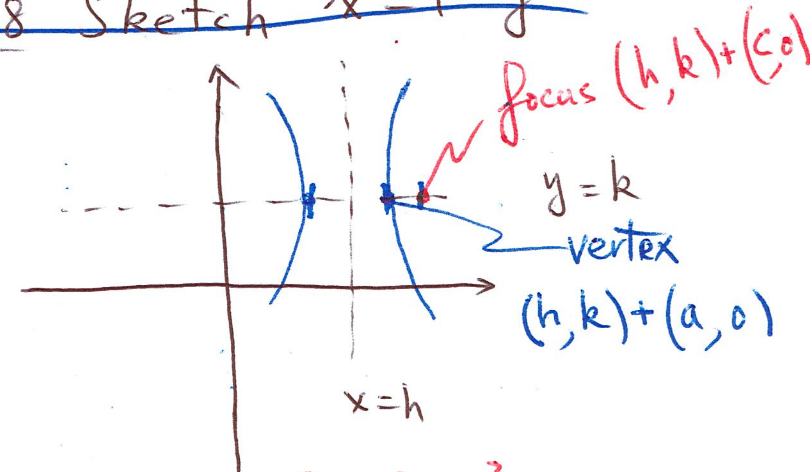
$$(x-h)^2 = 4p\,(y-k)$$

**hyperbola** $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

~~Ex. 7 Sketch $y = 2x^2 - 4x + 1$~~




~~Ex. 8 Sketch $x = 1 - y^2$~~




$c^2 = a^2 + b^2$

Find the vertex, focus, and directrix of the parabola.

$$\underline{y^2+2y}+8x+17=0$$

$$+1-1$$

$$(y+1)^2=-8x-16=-8(x+2)=4\cdot(-2)(x+2)$$

<u>vertex</u> $(h,k)=(-2,-1)$ $\qquad p=-2$

<u>focus</u> $(-2,-1)+(p,0)=(-2,-1)+(-2,0)=(-4,+1)$

<u>directrix</u> $x=h-p=-2+2=0$

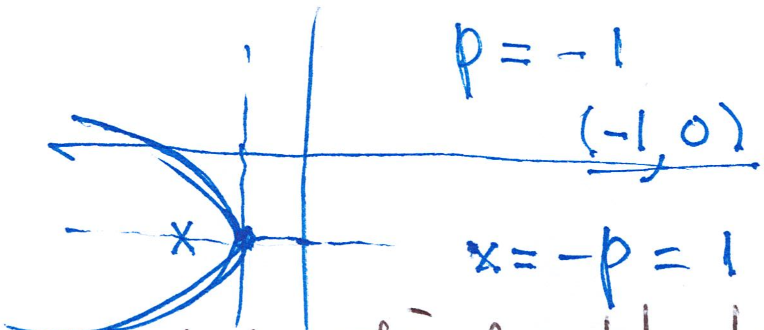
## 27. conic sections and shifted versions

(1) Find the vertex, focus, and directrix of the parabola given by $y^2+4y+4x+8=0$

$$0 = y^2+4y+4-4+4x+8 = (y+2)^2+4x+4$$

$$(y+2)^2 = -4\,(x+1)$$

vertex $(-1, -2)$



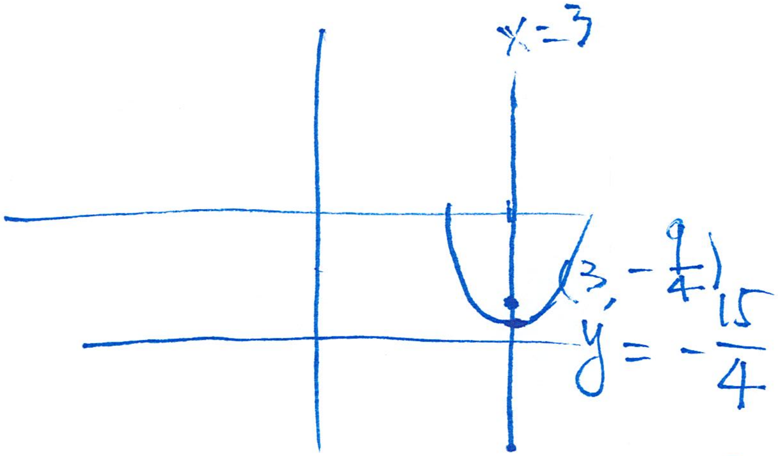

$p = -1$

$x = -p = 1$

focus $(-1,\ 0) + (-1, -2) = (-2, -2)$

directrix $x = 1 - 1 = 0$

(2) Find the equation of parabola whose focus is $(3, -\frac{9}{4})$ with the directrix $y = -\frac{15}{4}$.




$= -p$

$p = \frac{15}{4}$

$$(h,k) = \left(3, \frac{1}{2}\left(-\frac{9}{4} - \frac{15}{4}\right)\right) =$$

$$= \left(3, -\frac{\cancel{16}\ 24}{8}\right) = (3, -3)$$

$$(x-3)^2 = 4p(y+3) = 15(y+3)$$

(3) Find the vertices and foci of the ellipse given by $16x^2 - 32x + 4y^2 + 4y = 47$.

what is the value of $\underbrace{|PF_1| + |PF_2|}_{}$ $= 2a = 8$ where P is a point on the ellipse and $F_1$ and $F_2$ are the foci?

$$16(x-1)^2 + 4\left(y+\tfrac{1}{2}\right)^2 - 16 - 1 = 47 \qquad (h,k) = \left(1, -\tfrac{1}{2}\right)$$

$$\frac{(x-1)^2}{1} + \frac{\left(y+\frac{1}{2}\right)^2}{4} = \cancel{\frac{36}{16}} \; \frac{64}{16} = 4 \qquad a = 4, \quad b = 2, \quad c = \sqrt{a^2 - b^2}$$

$$= \sqrt{16-4}$$

$$= 2\sqrt{3}$$

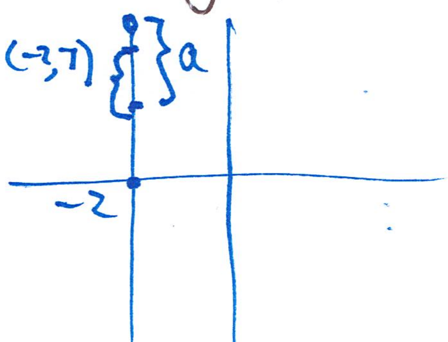
$$\frac{(x-1)^2}{4} + \frac{\left(y+\frac{1}{2}\right)^2}{16} = 1$$

vertices $\left(1, -\tfrac{1}{2}\right) \pm (0, 4)$

$= \left(1, -\tfrac{1}{2} \pm 4\right)$

foci $\left(1, -\tfrac{1}{2}\right) \pm \left(0, 2\sqrt{3}\right)$

(4) Find an equation of the ellipse whose vertices are $(-2,0)$ and $(-2,8)$, with one of the foci being $(-2,7)$.




$$(h,k) = \frac{1}{2}\left[(-2,0) + (-2,8)\right] = (-2, 4)$$

$$a = 8 - 4 = 4,$$

$$c = 7 - 4 = 3, \qquad b = \sqrt{a^2 - c^2} = \sqrt{16 - 9} = \sqrt{7}$$

$$\frac{(x+2)^2}{(\sqrt{7})^2} + \frac{(y-4)^2}{4^2} = 1$$

(5) Find the vertices, the foci, and the asymptotes of the hyperbola given by $4x^2 - y^2 - 24x - 6y + 43 = 0$

What is the value of $|PF_1 - PF_2| = 2a = 8$ where P is a point on the hyperbola and $F_1$ and $F_2$ are the two foci?

$$0 = 4(x^2 - 6x) - (y^2 + 6y) + 43$$

$$\cancel{= 4(x-3)^2} - (y+3)^2 \boxed{-36 + 9 + 43} + 16$$

$$\boxed{\frac{(y+3)^2}{16} - \frac{(x-3)^2}{4} = 1}$$

$(h, k) = (3, -3)$

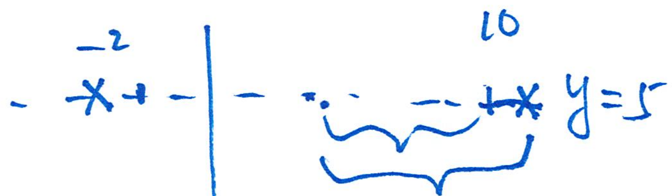
$$\begin{cases} \dfrac{y+3}{4} + \dfrac{x-3}{2} = 0 \\ \dfrac{y+3}{4} - \dfrac{x-3}{2} = 0 \end{cases}$$

$a = 4$, $b = 2$, $c = \sqrt{a^2 + b^2} = \sqrt{16 + 4} = 2\sqrt{5}$

vertices $(3, -3) \pm (0, 4)$, foci $(3, -3) \pm (0, 2\sqrt{5})$

(6) Find an equation of the hyperbola whose vertices are $(-2, 5)$ and $(10, 5)$ and whose foci are $(-3, 5)$ and $(11, 5)$.

$(h, k) = \frac{1}{2}[(-2, 5) + (10, 5)] = (4, 5)$




$a = 10 - 4 = 6$

$$\boxed{\cancel{c = \frac{1}{2}(-3 + 11) = 4}}$$

$c = 11 - 4 = 7$, $b = \sqrt{c^2 - a^2} = \sqrt{49 - 36} = \sqrt{13}$

$$\frac{(x-4)^2}{6^2} - \frac{(y-5)^2}{13} = 1$$