MATH 174 Quiz 1

Name: Solution Section: PID:

Solve the problem systematically and neatly and show all your work.

1.(2pts) Find the center and radius of the sphere

$$x^2 + y^2 + z^2 - 2x + 4y - 6z = 11.$$

Sol: $x^2 - 2x + y^2 + 4y + z^2 - 6z = 11$

$(x^2 - 2x + 1) + (y^2 + 4y + 4) + (z^2 - 6z + 9) = 1 + 4 + 9 + 11 = 25$

$(x-1)^2 + (y+2)^2 + (z-3)^2 = 25$

$\boxed{\text{Center } (1, -2, 3) \quad \text{radius} = 5}$

2. Let $P = (1, 2, 1)$ and $Q = (-3, 0, 5)$,

(1pt) (a) find the component form of $\overrightarrow{PQ}$.

$\overrightarrow{PQ} = \langle -3-1, 0-2, 5-1 \rangle = \boxed{\langle -4, -2, 4 \rangle}$

(1pt) (b) express $\overrightarrow{PQ}$ in the form of $v_1 \vec{i} + v_2 \vec{j} + v_3 \vec{k}$.

$\overrightarrow{PQ} = \boxed{-4\vec{i} - 2\vec{j} + 4\vec{k}}$

(2pts) (c) express $\overrightarrow{PQ}$ as a product of its length and the direction.

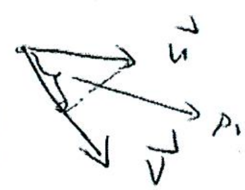
$|\overrightarrow{PQ}| = \sqrt{(-4)^2 + (-2)^2 + 4^2} = \sqrt{36} = 6$

$\overrightarrow{PQ} = |\overrightarrow{PQ}| * \dfrac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = 6 * \left(\dfrac{-4\vec{i} - 2\vec{j} + 4\vec{k}}{6} \right) = \boxed{6 * (-\tfrac{2}{3}\vec{i} - \tfrac{1}{3}\vec{j} + \tfrac{2}{3}\vec{k}}$

3. Let $\vec{u} = -2i + 3j - k$ and $\vec{v} = i + 2j - 3k$,

(2pts) (a) write $\vec{u}$ as the sum of a vector parallel to $\vec{v}$ and a vector orthogonal to $\vec{v}$.

Sol: $\text{Proj}_{\vec{v}} \vec{u} = \left(\dfrac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \right) \vec{v} = \dfrac{-2+6+3}{\left(\sqrt{1^2 + 2^2 + (-3)^2}\right)^2} (i + 2j - 3k) = \frac{1}{2}\vec{i} + \vec{j} - \frac{3}{2}\vec{k}$



$\vec{u} - \text{Proj}_{\vec{v}} \vec{u} = -2\vec{i} + 3\vec{j} - \vec{k} - \left(\frac{1}{2}\vec{i} + \vec{j} - \frac{3}{2}\vec{k} \right) = -\frac{5}{2}\vec{i} + 2\vec{j} + \frac{1}{2}\vec{k}$

$\vec{u} = \boxed{\left(\frac{1}{2}\vec{i} + \vec{j} - \frac{3}{2}\vec{k} \right) + \left(-\frac{5}{2}\vec{i} + 2\vec{j} + \frac{1}{2}\vec{k} \right)}$

(2pts) (b) find the angle between $\vec{u}$ and $\vec{v}$.

$\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{|\vec{u}|\,|\vec{v}|} = \dfrac{-2(1) + 3(2) + (-1)(3)}{\sqrt{(-2)^2 + 3^2 + (-1)^2} \cdot \sqrt{1^2 + 2^2 + (-3)^2}} = \dfrac{7}{14} = \dfrac{1}{2}$

$\theta = \cos^{-1}\left(\frac{1}{2}\right) = \boxed{60^\circ \text{ or } \frac{\pi}{3} \text{ radians}}$