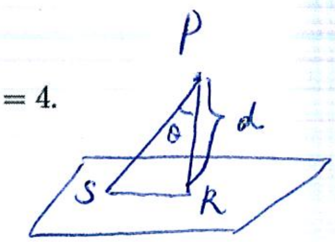
MATH 174 Quiz 2

Name: Solution PID: ____________

Solve the problem systematically and neatly and show all your work.

1.(2pts) Find the distance from point $P = (1,2,3)$ to the plane $2x + y + 2z = 4$.



Sol: choose a point S on the plane. let S be $(1,0,1)$

$\vec{PS} = \langle 0, -2, -2 \rangle \qquad \vec{n} = \langle 2, 1, 2 \rangle$

$$\Rightarrow d = \frac{|\vec{PS}\cdot\vec{n}|}{|\vec{n}|} = \frac{|0-2-4|}{\sqrt{2^2+1^2+2^2}} = \frac{6}{3} = \boxed{2}$$

2. Let $P = (1,2,3)$, $Q = (0,4,5)$ and $R = (-2,6,8)$,

(2pts) (a) find a unit vector which is perpendicular to plane PQR.

(2pts) (b) find the area of triangle PQR.

(2pts) (c) find the equation of the plane through points P, Q and R.

Sol: (a) $\vec{PQ} = \langle -1, 2, 2 \rangle$, $\vec{PR} = \langle -3, 4, 5 \rangle$

$$\Rightarrow \vec{PQ}\times\vec{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 2 \\ -3 & 4 & 5 \end{vmatrix} = 2\vec{i} - \vec{j} + 2\vec{k}$$

$$\Rightarrow \vec{n}_0 = \frac{\vec{PQ}\times\vec{PR}}{|\vec{PQ}\times\vec{PR}|} = \boxed{\frac{2}{3}\vec{i} - \frac{1}{3}\vec{j} + \frac{2}{3}\vec{k}}$$

$\left(-\frac{2}{3}\vec{i} + \frac{1}{3}\vec{j} - \frac{2}{3}\vec{k}\right.$ is also correct!$\left.\right)$

(b) area of $\triangle PQR = \frac{1}{2} *$ area of parrellogram by $\vec{PQ}$ & $\vec{PR}$ $= \frac{1}{2}|\vec{PQ}\times\vec{PR}| = \boxed{\frac{3}{2}}$

(c) $\vec{n} = 2\vec{i} - \vec{j} + 2\vec{k}$, point $P = (1,2,3)$ lies on the plane

$$\Rightarrow 2(x-1) - (y-2) + 2(z-3) = 0$$

$$\Rightarrow \boxed{2x - y + 2z = 6}$$

3.(2pts) Let $P = (1,3,0)$ and $Q = (2,2,2)$, find the parametric equation for the line through points P and Q.

Sol: $\vec{PQ} = \langle 1, -1, 2 \rangle$ point $P\ (1,3,0)$ | point $Q\ (2,2,2)$

$$\Rightarrow \boxed{\begin{aligned} x &= 1 + 1*t = 1+t \\ y &= 3 + (-1)*t = 3-t \\ z &= 0 + 2*t = 2t \end{aligned}} \quad -\infty < t < \infty$$

or $\boxed{\begin{aligned} x &= 1-t \\ y &= 3+t \\ z &= -2t \end{aligned}}$

$$\Rightarrow \boxed{\begin{aligned} x &= 2 \pm t \\ y &= 2 \mp t \\ z &= 2 \pm 2t \end{aligned}} \quad -\infty < t < \infty$$