MATH 174 Quiz 3

Name: Solution PID: ______________

Solve the problem systematically and neatly and show all your work.

1. Let $\mathbf{r}(t) = (e^t)\mathbf{i} + (\sin t)\mathbf{j} + (\cos t)\mathbf{k}$ be the position of a particle in space at time t,

(2pts) (a) Find the parametric equation of the line tangent to the curve $\mathbf{r}(t)$ at $t = 0$.

(2pts) (b) Evaluate $\int_0^{\pi} [(e^t)\mathbf{i} + (\sin t)\mathbf{j} + (\cos t)\mathbf{k}]\, dt$.

Sol. (a) $\vec{v}(t) = e^t \vec{i} + \cos t \vec{j} - \sin t \vec{k} \Rightarrow \vec{v}(0) = \langle e^0, \cos 0, -\sin 0 \rangle$
$= \langle 1, 1, 0 \rangle$

point $P = \vec{r}(0) = (e^0, \sin 0, \cos 0) = (1, 0, 1)$

$\Rightarrow \underline{x = 1+t \quad y = 0 + t = t \quad z = 1 + 0t = 1} \quad -\infty < t < \infty$

(b) $\int_0^{\pi} (e^t \vec{i} + \sin t \vec{j} + \cos t \vec{k})\, dt = e^t \vec{i} - \cos t \vec{j} + \sin t \vec{k} \Big]_0^{\pi}$

$= (e^{\pi} - e^0)\vec{i} - (\cos \pi - \cos 0)\vec{j} + (\sin \pi - \sin 0)\vec{k}$

$= \boxed{(e^{\pi} - 1)\vec{i} + 2\vec{j}}$

2. (3pts) Find the arc length of the curve $\mathbf{r}(t) = (t\sin t + \cos t)\mathbf{i} + (t\cos t - \sin t)\mathbf{j}$ from $(1, 0)$ to $(-1, -\pi)$.

Sol: $(1, 0) \Rightarrow t = 0 \qquad (-1, -\pi) \Rightarrow t = \pi$

$\vec{v}(t) = \frac{d\vec{r}}{dt} = t\cos t \vec{i} - t\sin t \vec{j} \Rightarrow |\vec{v}| = t$

$\Rightarrow s = \int_0^{\pi} |\vec{v}|\, dt = \int_0^{\pi} t\, dt = \frac{1}{2} t^2 \Big|_0^{\pi} = \boxed{\frac{1}{2}\pi^2}$

3.(3pts) A golf ball leaves the ground at a $30°$ angle at a speed of 90 ft/sec. Will it clear the top of a 30−ft tree that is in the way, 135 ft down the fairway? Explain.

Sol: $\boxed{\text{No}}$ $\quad v_0 = 90 \quad \alpha = 30° \quad g = 32\ ft/s^2$

$\vec{r}(t) = v_0 \cos\alpha t \vec{i} + (v_0 \sin\alpha t - \frac{1}{2} g t^2)\vec{j} = 45\sqrt{3}\, t \vec{i} + (45t - 16t^2)\vec{j}$

$x = 45\sqrt{3}\, t \qquad$ if $x = 135 \Rightarrow t = \sqrt{3}$ sec

$y = 45t - 16t^2 \qquad \Rightarrow y(\sqrt{3}) = 45\sqrt{3} - 16(\sqrt{3})^2 = \boxed{29.94\ ft}$

$< 30\ ft$

$\Rightarrow$ The golf ball will not clear the top of the tree.