MATH 174 Quiz 4

Name: Solution PID: _____

1.(6pts) Let $\mathbf{r}(t) = (3\sin t)\mathbf{i} + (3\cos t)\mathbf{j} + 4t\mathbf{k}$ be the position of a particle in space at time $t$, find the binormal vector $\mathbf{B}$, the curvature $\kappa$ and the torsion $\tau$.

Sol: $\vec{v}(t) = 3\cos t\,\vec{i} - 3\sin t\,\vec{j} + 4\vec{k} \Rightarrow |\vec{v}| = 5$

$\vec{T}(t) = \frac{\vec{v}}{|\vec{v}|} = \frac{3}{5}\cos t\,\vec{i} - \frac{3}{5}\sin t\,\vec{j} + \frac{4}{5}\vec{k}$

$\frac{d\vec{T}}{dt} = -\frac{3}{5}\sin t\,\vec{i} - \frac{3}{5}\cos t\,\vec{j} + 0\vec{k} \Rightarrow \left|\frac{d\vec{T}}{dt}\right| = \frac{3}{5}$

$\Rightarrow \boxed{K = \frac{|d\vec{T}/dt|}{|\vec{v}|} = \frac{3/5}{5} = \frac{3}{25}}$

$\vec{N} = \frac{d\vec{T}/dt}{|d\vec{T}/dt|} = -\sin t\,\vec{i} - \cos t\,\vec{j}$

$\Rightarrow \vec{B} = \vec{T} \times \vec{N} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{3}{5}\cos t & -\frac{3}{5}\sin t & \frac{4}{5} \\ -\sin t & -\cos t & 0 \end{vmatrix} = \boxed{\frac{4}{5}\cos t\,\vec{i} - \frac{4}{5}\sin t\,\vec{j} - \frac{3}{5}\vec{k}}$

$\frac{d\vec{B}}{dt} = -\frac{4}{5}\sin t\,\vec{i} - \frac{4}{5}\cos t\,\vec{j} \Rightarrow \frac{d\vec{B}}{dt} \cdot \vec{N} = \frac{4}{5}\sin^2 t + \frac{4}{5}\cos^2 t = \frac{4}{5}$

$\Rightarrow \tau = -\frac{\frac{d\vec{B}}{dt} \cdot \vec{N}}{|\vec{v}|} = -\frac{4/5}{5} = \boxed{-\frac{4}{25}}$

2.(4pts) Let $\mathbf{r}(t) = (t^2+3)\mathbf{i} + (t + \frac{1}{3}t^3 + 1)\mathbf{j} + (t - \frac{1}{3}t^3 - 1)\mathbf{k}$, write $\mathbf{a}$ in the form of $\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$ without finding $\mathbf{T}$ and $\mathbf{N}$.

Sol: $\vec{v}(t) = 2t\,\vec{i} + (1+t^2)\,\vec{j} + (1-t^2)\,\vec{k} \Rightarrow |\vec{v}| = \sqrt{2}(t^2+1)$

$\vec{a}(t) = 2\vec{i} + 2t\,\vec{j} - 2t\,\vec{k} \Rightarrow |\vec{a}|^2 = 8t^2 + 4$

$a_T = \frac{d|\vec{v}|}{dt} = 2\sqrt{2}\,t$

$a_N = \sqrt{|\vec{a}|^2 - a_T^2} = \sqrt{8t^2 + 4 - 8t^2} = \sqrt{4} = 2$

$\Rightarrow \boxed{\vec{a} = 2\sqrt{2}\,\vec{T} + 2\vec{N}}$