MATH 174 Quiz 5

Name: Solution PID: ____________

Solve the problem systematically and neatly and show all your work.

1.(2pts) Use the formula for implicit differentiation to find $\frac{dy}{dx}$ if $xe^y - \sin(xy) + x^2 - 2y = 5$.

Sol: Let $F(x,y) = xe^y - \sin(xy) + x^2 - 2y - 5 = 0$

$$\Rightarrow \frac{dy}{dx} = -\frac{F_x}{F_y} = \boxed{-\frac{e^y - y\cos(xy) + 2x}{xe^y - x\cos(xy) - 2}}$$

2.(2pts) Let $w = xy + y^2 - 2$, $x = uv$ and $y = \frac{u}{v}$, use chain rule to evaluate $\frac{\partial w}{\partial v}$ at $(2, \frac{1}{2})$.

Sol: $\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial v}$

$\frac{\partial w}{\partial x} = y$ $\quad \frac{\partial w}{\partial y} = x + 2y$

$\frac{\partial x}{\partial v} = u$ $\quad \frac{\partial y}{\partial v} = -\frac{u}{v^2}$

$$\Rightarrow \frac{\partial w}{\partial v} = y \cdot u + (x+2y) \cdot \left(-\frac{u}{v^2}\right) = \frac{u}{v} \cdot u + \left(uv + \frac{2u}{v}\right) \cdot \left(-\frac{u}{v^2}\right)$$

$$\Rightarrow \frac{\partial w}{\partial v}\Big|_{(2,\frac{1}{2})} = \frac{2}{\frac{1}{2}} \cdot 2 + \left(2 \cdot \frac{1}{2} + \frac{2 \cdot 2}{\frac{1}{2}}\right) \cdot \left(-\frac{2}{(\frac{1}{2})^2}\right) = \boxed{-64}$$

3. Let $f(x,y,z) = x^2y^3z^6$,

(2pts) (a) find ∇f at $P_0(1,1,1)$.

(2pts) (b) find the directional derivative of $f(x,y,z)$ in the direction of ∇f at $P_0(1,1,1)$. $\quad \vec{u} = \frac{\vec{\nabla f}}{|\vec{\nabla f}|}$

(2pts) (c) find the normal line of the surface $f(x,y,z) = x^2y^3z^6$ at $P_0(1,1,1)$.

Sol: (a) $\vec{\nabla f} = \langle f_x, f_y, f_z \rangle = \langle 2xy^3z^6, 3x^2y^2z^6, 6x^2y^3z^5 \rangle$

$$\Rightarrow \vec{\nabla f}\Big|_{(1,1,1)} = \boxed{\langle 2, 3, 6 \rangle}$$

(b) $$\left(\frac{df}{ds}\right)_{\frac{\vec{\nabla f}}{|\vec{\nabla f}|}, P_0} = \vec{\nabla f}\Big|_{P_0} \cdot \frac{\vec{\nabla f}}{|\vec{\nabla f}|}\Big|_{P_0} = \langle 2,3,6 \rangle \cdot \frac{\langle 2,3,6 \rangle}{\sqrt{2^2+3^2+6^2}} = \boxed{7}$$

(c) $$\boxed{\begin{aligned} x &= 1 + 2t \\ y &= 1 + 3t \\ z &= 1 + 6t \end{aligned} \quad -\infty < t < \infty}$$

point $(1,1,1)$, direction $\langle 2,3,6 \rangle$