MATH 174 Quiz 6

Name: Solution PID: ____________________

1. Let $z = f(x, y) = \sqrt{x^2 + y}$,

(2pts) (a) find an equation for the tangent plane of $z = f(x, y)$ at $(4, 9)$.

(2pts) (b) use the linearization $L(x, y)$ of the function $f(x, y)$ at $(4, 9)$ to approximate $f(5, 8)$.

Sol: let $F(x,y,z) = f(x,y) - z \Rightarrow F_x = f_x(x,y) = \frac{2x}{2\sqrt{x^2+y}}$ $F_y = \frac{1}{2\sqrt{x^2+y}}$ $F_z = -1$

$z = f(4,9) = \sqrt{25} = 5$ $F_x|_{(4,9)} = \frac{4}{5}$ $F_y|_{(4,9)} = \frac{1}{10}$

$\Rightarrow$ (a) tangent plane is $\boxed{\frac{4}{5}(x-4) + \frac{1}{10}(y-9) - (z-5) = 0}$

(b) $L(x,y) = f(4,9) + f_x(4,9)(x-4) + f_y(4,9)(y-9) = 5 + \frac{4}{5}(x-4) + \frac{1}{10}(y-9)$

$\Rightarrow f(5,8) \approx L(5,8) = 5 + \frac{4}{5} \times 1 + \frac{1}{10} \times (-1) = \boxed{5.7}$

(3pts) 2. Find all the critical points of the function $f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy$ and identify them as the local maxima, local maxima or saddle points.

Sol: $\left.\begin{array}{l} f_x = 12x - 6x^2 + 6y = 0 \\ f_y = 6y + 6x = 0 \Rightarrow x = -y \end{array}\right\} \Rightarrow 12x - 6x^2 - 6x = 0 \Rightarrow x = 0$ or $x = 1$

$\Rightarrow y = 0$ or $y = -1$

critical points are $(0,0)$ and $(1,-1)$

$f_{xx} = 12 - 12x$ $H(x,y) = f_{xx}f_{yy} - f_{xy}^2 = (12-12x) * 6 - 6^2$

$f_{yy} = 6$

$f_{xy} = 6$

at $(0,0)$ $H(0,0) = 36 > 0 \Rightarrow$ $(0,0)$ is a local MIN

at $(1,-1)$ $H(1,-1) = -36 < 0 \Rightarrow$ $(1,-1)$ is a saddle point

(3pts) 3. Find absolute maximum and minimum values of $f(x, y) = x^2 + 2y^2 - 4x$ on the disc $x^2 + y^2 \le 1$.

Sol: ① interior points $x^2+y^2<1$

look for critical points

$\left.\begin{array}{l} f_x = 2x - 4 = 0 \\ f_y = 4y = 0 \end{array}\right\} \Rightarrow \begin{array}{l} x = 2 \\ y = 0 \end{array}$

But $2^2 + 0^2 = 4 > 1$

$\Rightarrow (2,0)$ is NOT an interior pt

$\Rightarrow$ it has no critical point inside the disk $x^2+y^2 \le 1$

② Boundary points $x^2+y^2=1 \Rightarrow y^2 = 1-x^2$

$f(x,y) = x^2 + 2y^2 - 4x = x^2 + 2(1-x^2) - 4x$

$= -x^2 - 4x + 2$

let $g(x) = -x^2 - 4x + 2 \Rightarrow g'(x) = -2x - 4 = 0$

$\Rightarrow \left.\begin{array}{l} x = -2 \\ x^2+y^2=1 \end{array}\right\} \Rightarrow$ no solution for y

$\Rightarrow g(x)$ has no critical pt. on the circle $x^2+y^2=1$

$g(x) = -x^2 - 4x + 2 = -(x+2)^2 + 6$

$\Rightarrow g_{max} = g(-1) = -(-1+2)^2 + 6 = 5 \Rightarrow \boxed{f_{max} = f(-1,0) = 5}$

$g_{min} = g(1) = -(1+2)^2 + 6 = -3 \Rightarrow \boxed{f_{min} = f(1,0) = -3}$