MATH 174 Quiz 7

Name: Solution PID: ____________

(3pts) 1. Use Taylor's formula to find a quadratic approximation of $e^x \cos y$ at the origin $(0,0)$.

Sol: $f(x,y) = e^x \cos y$

$f_x(x,y) = e^x \cos y$

$f_y(x,y) = -e^x \sin y$

$f_{xx}(x,y) = e^x \cos y$

$f_{xy}(x,y) = -e^x \sin y$

$f_{yy}(x,y) = -e^x \cos y$

$\Rightarrow f(0,0) = 1 = f_x(0,0) = f_{xx}(0,0)$

$f_y(0,0) = f_{xy}(0,0) = 0$

$f_{yy}(0,0) = -1$

$$\Rightarrow P_2(x,y) = f(0,0) + f_x(0,0)x + f_y(0,0)y + \frac{1}{2}\left[f_{xx}(0,0)x^2 + 2f_{xy}(0,0)xy + f_{yy}(0,0)y^2\right]$$

$$= \boxed{1 + x + \frac{1}{2}(x^2 - y^2)}$$

(3pts) 2. Find $(\frac{\partial w}{\partial y})_x$ at $(w,x,y,z) = (4,2,1,-1)$ if

$$w = x^2y^2 + yz - z^3, x^2 + y^2 + z^2 = 6$$

Sol: $(\frac{\partial w}{\partial y})_x$ implies x and y are independent variables

$$\frac{\partial w}{\partial y} = 2x^2y + z + y\frac{\partial z}{\partial y} - 3z^2\frac{\partial z}{\partial y}$$

$$2y + 2z\frac{\partial z}{\partial y} = 0 \Rightarrow \frac{\partial z}{\partial y} = -\frac{y}{z}$$

$$\Rightarrow \left(\frac{\partial w}{\partial y}\right)_x = 2x^2y + z - (y - 3z^2)(-\frac{y}{z})$$

$$= \boxed{2x^2y + z + \frac{y^2}{z} - 3yz}$$

(4pts) 3. Use the method of Lagrange Multiplier to find the maximum of $f(x,y) = x^2 + y^2$ subject to the constraint $x^2 - 2x + y^2 - 4y = 0$.

Sol: Need to solve $\vec{\nabla} f = \lambda \vec{\nabla} g \Rightarrow \langle 2x, 2y\rangle = \lambda\langle 2x-2, 2y-4\rangle$

with $g(x,y) = x^2 - 2x + y^2 - 4y = 0$

$\Rightarrow 2x = 2\lambda x - 2\lambda \Rightarrow 2\lambda = 2\lambda x - 2x$

$2y = 2\lambda y - 4\lambda \Rightarrow 4\lambda = 2\lambda y - 2y$

$\Rightarrow (2\lambda - 2)x = (\lambda - 1)y$

$\Rightarrow (\lambda - 1)(2x - y) = 0$

Case I $\lambda = 1 \Rightarrow 2x = 2x - 2$ No solution

Case II $2x - y = 0 \Rightarrow y = 2x$

$x^2 - 2x + y^2 - 4y = 0$

$\Rightarrow 5x^2 = 10x \Rightarrow \boxed{x=0, y=0}$ or $\boxed{x=2, y=4}$

$f(0,0) = 0$ $f(2,4) = 20 \Rightarrow \boxed{f_{max} = 20}$