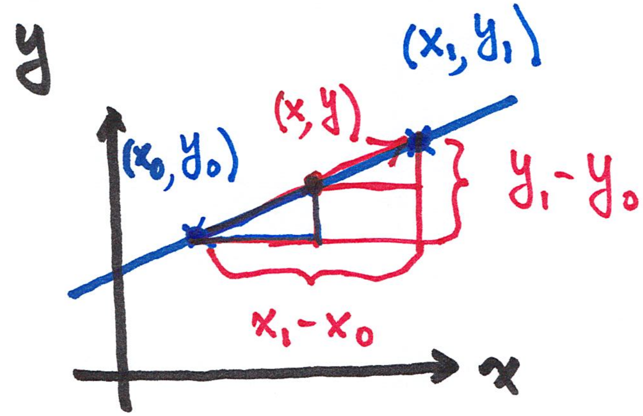
# §1.5 Equations of Lines and Planes

## Line in $R^2$

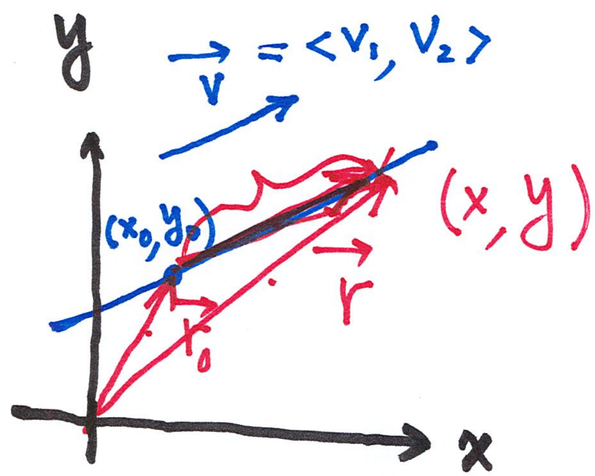



$$\frac{y - y_0}{y_1 - y_0} = \frac{x - x_0}{x_1 - x_0}.$$

$$\vec{v} = \langle x_1 - x_0,\ y_1 - y_0 \rangle$$

Given $(x_0, y_0) \in \mathcal{L}$ $\qquad \vec{r}_0 = \langle x_0, y_0 \rangle$

$\vec{v} \parallel \mathcal{L}$ $\qquad$ vector eq




$$\vec{r} = \langle x, y \rangle = \vec{r}_0 + \underline{t}\,\vec{v}$$

sym. eq.

$$\begin{cases} x = x_0 + t v_1 \\ y = y_0 + t v_2 \end{cases}$$

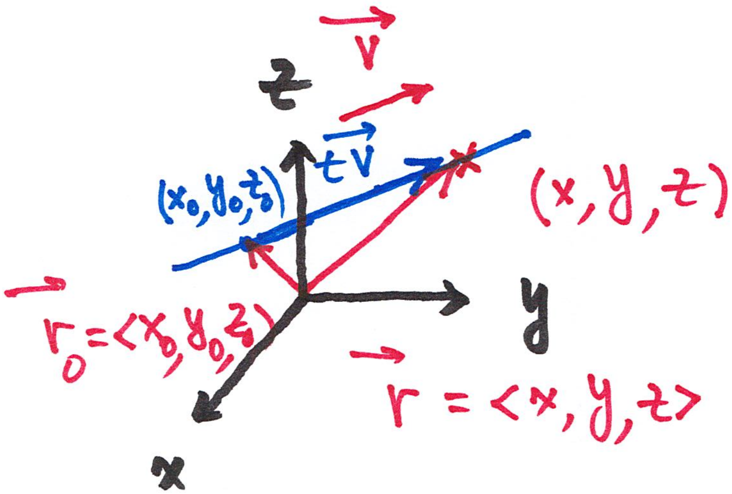
$\boxed{\text{parametric}}$ eq.

$$\frac{x - x_0}{v_1} = \frac{y - y_0}{v_2}$$

# Line in $\mathbb{R}^3$

Given a pt $(x_0, y_0, z_0) \in \mathcal{L}$

a vector $\vec{v} = \langle v_1, v_2, v_3 \rangle \parallel \mathcal{L}$




$$\vec{r} = \vec{r}_0 + t\vec{v} \qquad \text{vector eq.}$$

$$\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t \langle v_1, v_2, v_3 \rangle$$

$$= \langle x_0, y_0, z_0 \rangle + \langle tv_1, tv_2, tv_3 \rangle$$

$$= \langle x_0 + tv_1, y_0 + tv_2, z_0 + tv_3 \rangle$$

indep. var.

$$\begin{cases} x = x_0 + t v_1 \\ y = y_0 + t v_2 \\ z = z_0 + t v_3 \end{cases} \quad \text{parametric eq.}$$

$$\boxed{\frac{x - x_0}{v_1} = \frac{y - y_0}{v_2}} = \frac{z - z_0}{v_3}$$

sym eq.

$$ax + by + cz = d$$

#9 (P831) Find parametric and symmetric equations for the line through the points $(-8, 1, 4)$ and $(3, -2, 4)$

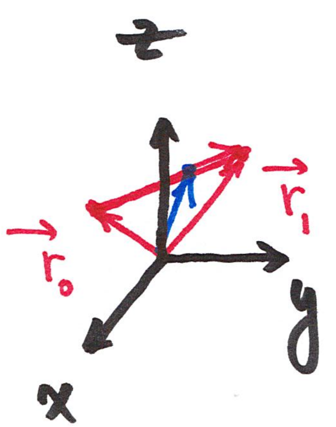
$\vec{r}_0 = \langle -8, 1, 4 \rangle$, $\vec{v} = \langle 11, -3, 0 \rangle$,

$$\begin{cases} x = -8 + 11t \\ y = 1 - 3t \\ z = 4 \end{cases}$$

#17 (P831) Find a vector equation for the line segment from $(6,-1,9)$ to $(7,6,0)$

$\vec{r}_0$, $\vec{v} = \langle 7, 6, 0 \rangle - \langle 6, -1, 9 \rangle = \langle 1, 7, -9 \rangle$

$= \vec{r}_1 - \vec{r}_0$




$$\vec{r} = \vec{r}_0 + t(\vec{r}_1 - \vec{r}_0) = (1-t)\vec{r}_0 + t\vec{r}_1$$

$t = 0 \Rightarrow \vec{r}_0$ $\quad 0 \le t \le 1$

$t = 1 \Rightarrow \vec{r}_1$

#3 (p831) Find vector and parametric equations for line through $(2, 2.4, 3.5)$ and parallel to $3\vec{i} + 2\vec{j} - \vec{k}$.
Also, find two other points on the line.

$\vec{r}_0 = \langle 2, 2.4, 3.5 \rangle$ , $\vec{v} = ? = \langle 3, 2, -1 \rangle$

V. $\vec{r} = \langle 2, 2.4, 3.5 \rangle + t \langle 3, 2, -1 \rangle$

P. $\begin{cases} x = 2 + 3t \\ y = 2.4 + 2t \\ z = 3.5 - t \end{cases}$

$\cancel{t = 0 \quad (2}$

$t = 1$ $(5, 4.4, 2.5)$

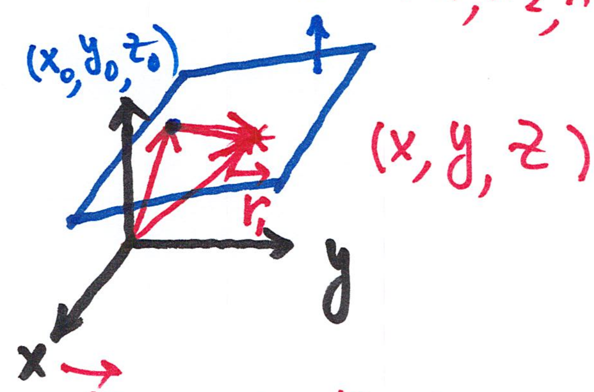
$t = -1$ $(-1, 0.4, 4.5)$

# Eq. of a Plane

**Given** a pt $(x_0, y_0, z_0) \in \mathcal{P}$

a vector $\vec{n} \perp \mathcal{P}$

$\vec{n} = \langle n_1, n_2, n_3 \rangle$

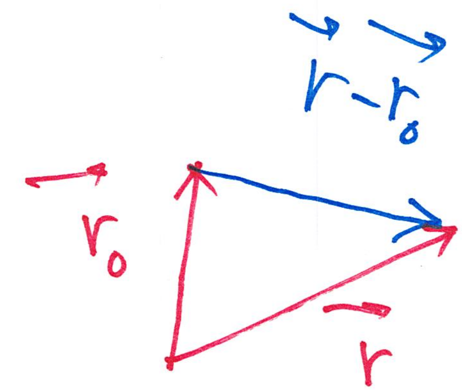



$\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$

$\vec{r} = \langle x, y, z \rangle$

$$\vec{r} - \vec{r}_0 \perp \vec{n}$$

$$\Longleftrightarrow \boxed{(\vec{r} - \vec{r}_0) \cdot \vec{n} = 0}$$




$$0 = \langle n_1, n_2, n_3 \rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle$$

$$= n_1 (x - x_0) + n_2 (y - y_0) + n_3 (z - z_0)$$

$$\underline{n_1 x + n_2 y + n_3 z = d} \equiv n_1 x_0 + n_2 y_0 + n_3 z_0$$

**#28 (P831)** Find an equation of the plane through the point $(3, -2, 8)$ and parallel to the plane $z = x + y$ $\Longrightarrow$ $x + y - z = 0$

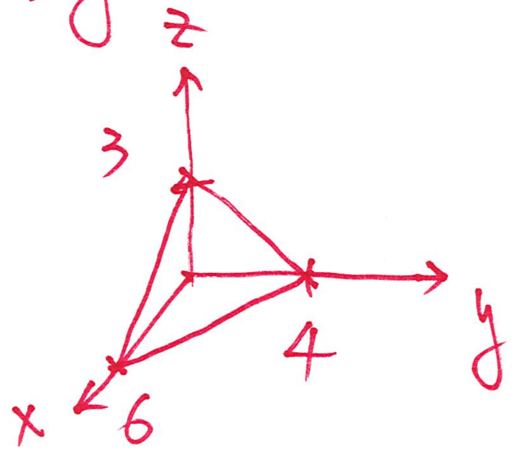
$$\left\{\begin{array}{l} \vec{n} = \langle 1, 1, -1 \rangle \\ (3, -2, 8) \end{array}\right. \Longrightarrow \quad x + y - z = 3 - 2 - 8 = -7$$

**Example 4** Find an equation of the plane through the point $(2, 4, -1)$ with normal vector $\vec{n} = \langle 2, 3, 4 \rangle$. Find the intercepts and sketch the plane.

$$2x + 3y + 4z = 2\cdot 2 + 3\cdot 4 + 4\cdot(-1) = 12$$

$3y = 12$

$2x = 12$




$4z = 12$