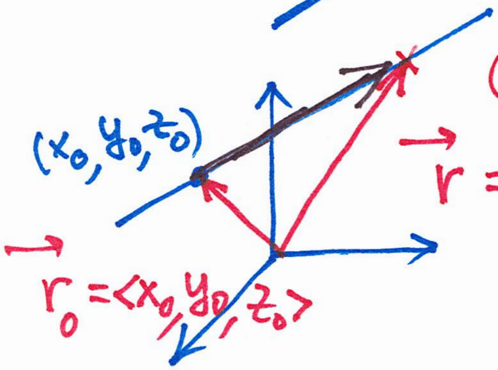
$$\vec{v} = \langle v_1, v_2, v_3 \rangle \parallel \mathcal{L}$$



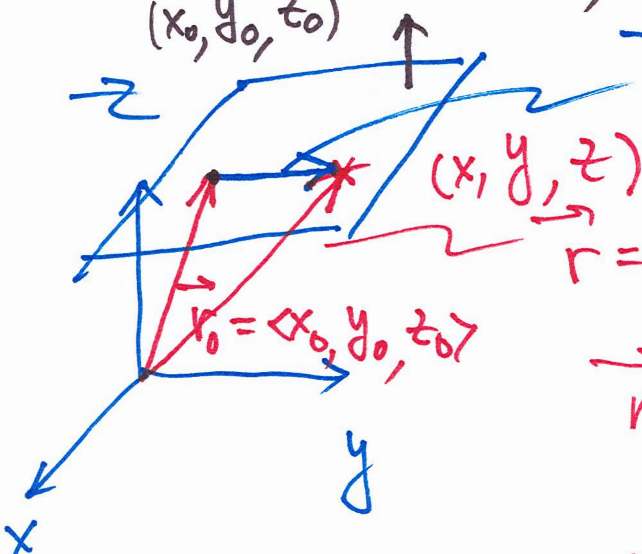
(x, y, z)

$$\vec{r} = \langle x, y, z \rangle = \vec{r}_0 + t\vec{v}$$

$$\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$$

$$\begin{cases} x = x_0 + t v_1 \\ y = y_0 + t v_2 \\ z = z_0 + t v_3 \end{cases}$$

$$\vec{n} = \langle n_1, n_2, n_3 \rangle \perp \mathcal{P}$$



$$\vec{r} - \vec{r}_0 = \langle x - x_0, y - y_0, z - z_0 \rangle$$

(x, y, z)

$$\vec{r} = \langle x, y, z \rangle$$

$$\vec{n} \perp \mathcal{P} \Longrightarrow \vec{n} \perp \vec{r} - \vec{r}_0$$

$$\Longleftrightarrow 0 = \vec{n} \cdot (\vec{r} - \vec{r}_0) = \langle n_1, n_2, n_3 \rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle$$

$$= n_1 (x - x_0) + n_2 (y - y_0) + n_3 (z - z_0)$$

#32 (P831) Find an equation of the plane through the points

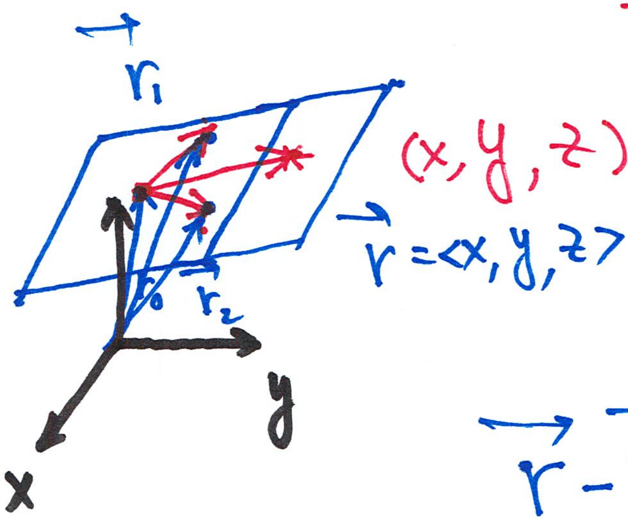
$(2,1,2)$, $(3,-8,6)$, and $(-2,-3,1)$

$(x_0, y_0, z_0) = (2,1,2)$

$$\vec{n} = \left(\langle 3,-8,6\rangle - \langle 2,1,2\rangle\right) \times \left(\langle -2,-3,1\rangle - \langle 2,1,2\rangle\right)$$

$$\begin{vmatrix} \vec{i} & \vec{j} & k \\ 1 & -9 & 4 \\ -4 & -4 & -1 \end{vmatrix}$$

$$= \underline{\underline{\langle 1,-9,4\rangle}} \times \underline{\underline{\langle -4,-4,-1\rangle}}$$



$$\langle x-2,\ y-1,\ z-2\rangle = \underline{\underline{u}}\langle 1,-9,4\rangle + \underline{\underline{v}}\langle -4,-4,-1\rangle$$

$$\vec{r} - \vec{r}_0 = u\left(\vec{r}_1 - \vec{r}_0\right) + v\left(\vec{r}_2 - \vec{r}_0\right)$$

Ex. 6 (P828) Find the point at which the line with parametric equations

$$\begin{cases} x = 2+3t \\ y = -4t \\ z = 5+t \end{cases}$$

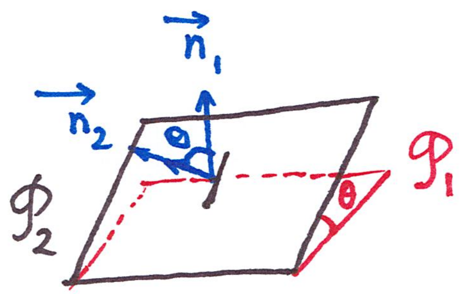
intersects the plane $4x + 5y - 2z = 18$

$$18 = 4(2+3t) + 5(-4t) - 2(5+t)$$
$$= (12-20-2)t + (8-10)$$
$$= -10t - 2$$
$$\Rightarrow -10t = 20 \Rightarrow t = -2$$
$$x = -4,\ y = 8,\ z = 3$$

$$\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{5}$$

$$\begin{cases} x = 1+t \\ y = -2+2t \\ z = 3+5t \end{cases}$$

Relation of two planes



(1) $P_1 \parallel P_2 \Longleftrightarrow \cancel{\vec{n}_1 \perp \vec{n}_2}$ $\vec{n}_1 \parallel \vec{n}_2$

(2) $P_1 \perp P_2 \Longrightarrow \vec{n}_1 \perp \vec{n}_2$

(3) the angle between two planes P_1 and P_2

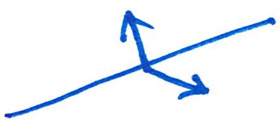
$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1|\,|\vec{n}_2|}$$

#54 (P832) Determine whether the planes: $1x - y + 3z = 1$ and $3x + y - z = 2$ are parallel, perpendicular, or neither. If neither, (a) find the angle between two planes, (b) find parametric equations for the line of intersection.

(a) $\vec{n}_1 = \langle 1, -1, 3 \rangle$ $\qquad \vec{n}_1 \not\parallel \vec{n}_2$

$\vec{n}_2 = \langle 3, 1, -1 \rangle$ $\qquad \vec{n}_1 \cdot \vec{n}_2 = \langle 1, -1, 3 \rangle \cdot \langle 3, 1, -1 \rangle$

$|\vec{n}_1| |\vec{n}_2| \cos\theta$ $\qquad = 3 - 1 - 3 = -1 \neq 0 \Rightarrow n_1 \not\perp n_2$



$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| \, |\vec{n}_2|} = \frac{-1}{\sqrt{1^2 + (-1)^2 + 3^2}\,\sqrt{3^2 + 1^2 + (-1)^2}} = \frac{-1}{\sqrt{11} \cdot \sqrt{11}} = -\frac{1}{11}$$

(b) pt $(x_0, y_0, z_0) \in \mathcal{L}$

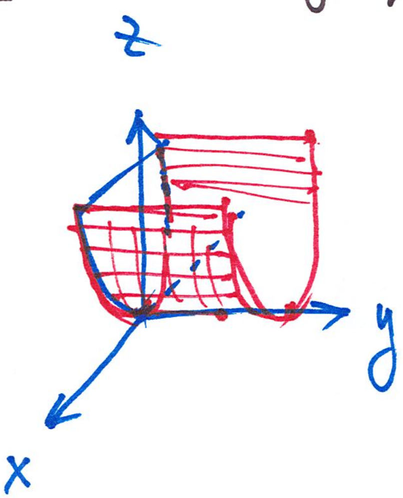
$$\begin{cases} x - y = 1 \\ 3x + y = 2 \end{cases} \qquad 4x = 3 \Rightarrow x = \frac{3}{4}, \; y = x - 1 = -\frac{1}{4} \qquad \left(\frac{3}{4}, -\frac{1}{4}, 0\right)$$

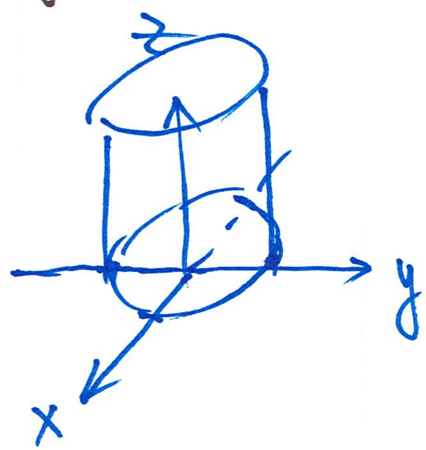
$$\vec{v} = \vec{n}_1 \times \vec{n}_2$$

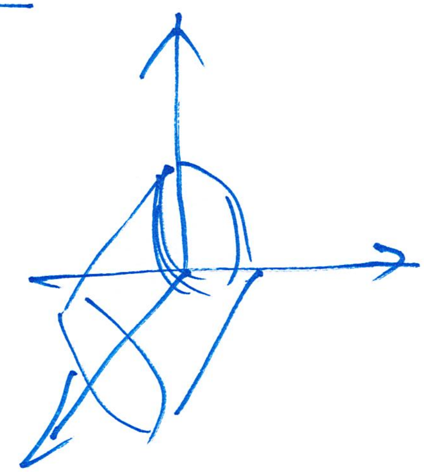
§12.6 Cylinders and Quadric Surfaces

Cylinders a surface that consists of all lines (called rulings) that are parallel to a given line and pass through a given plane curve.

Ex. 1-2 Sketch the graph of the surface (1) $z = x^2$, (2) $x^2 + y^2 = 1$, (3) $y^2 + z^2 = 1$







Quadric Surfaces

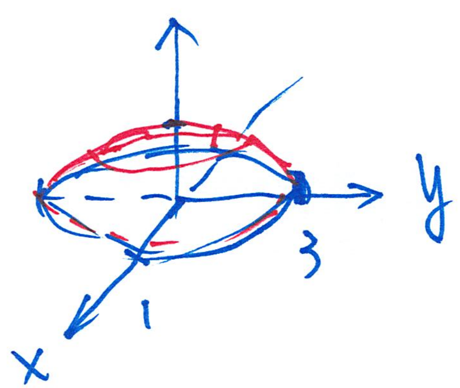
$$Ax^2+By^2+Cz^2+Dxy+Eyz+Fxz+Gx+Hy+Iz+J=0$$

standard forms

$$Ax^2+By^2+Cz^2+J=0 \quad \text{or} \quad Ax^2+By^2+Iz=0$$

Ex. 3 Use traces to sketch the quadric surface with equation: $x^2+\frac{y^2}{9}+\frac{z^2}{4}=1$

intersection between the surface with planes parallel to the coordinate planes



xy-plane $\quad z=0$

$$x^2+\frac{y^2}{9}=1$$

yz-plane $\quad x=0$

$$\frac{y^2}{9}+\frac{z^2}{4}=1$$