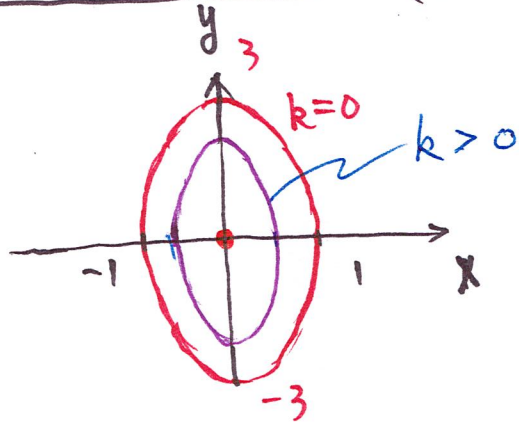


Quadric Surfaces

Ex. 3 Use traces to sketch

horizontal traces ($z=k$)



$$\underline{k=2} \quad x^2 + \frac{y^2}{9} = 0$$

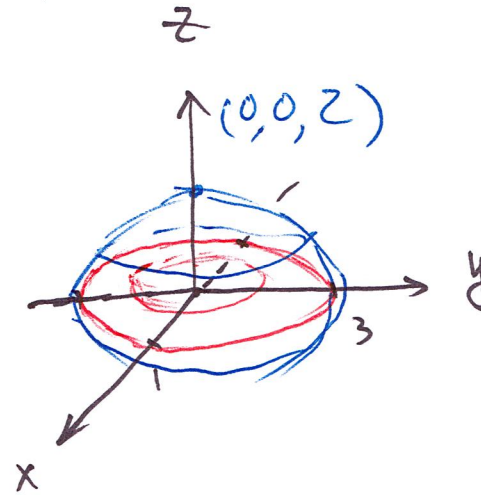
$$(0, 0, 2)$$

$$x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$$

$$x^2 + \frac{y^2}{9} = 1 - \frac{k^2}{4}$$

ellipsoid

$$1 - \frac{k^2}{4} \geq 0 \Rightarrow \underline{-2 \leq k \leq 2}$$

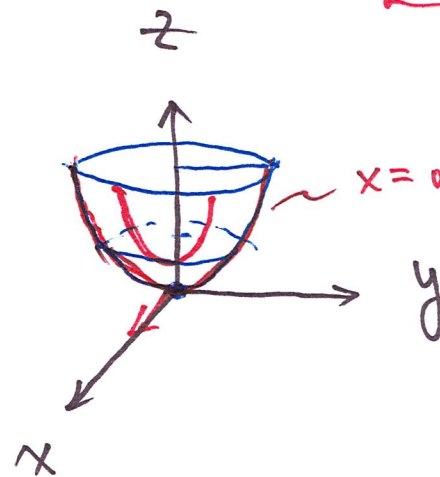
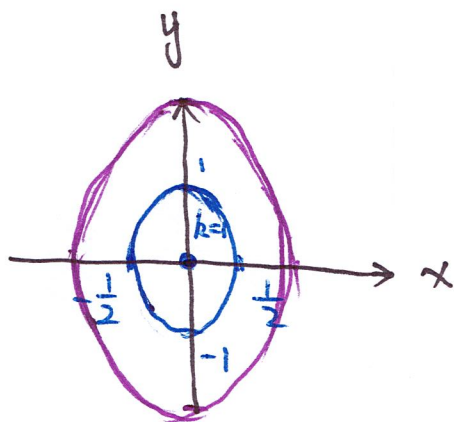


Ex. 4 Use traces to sketch the surface $z = 4x^2 + y^2$ elliptic
paraboloid

• $z = k \geq 0$

$$4x^2 + y^2 = k$$

$$4x^2 + y^2 = 1$$



• $x = k$

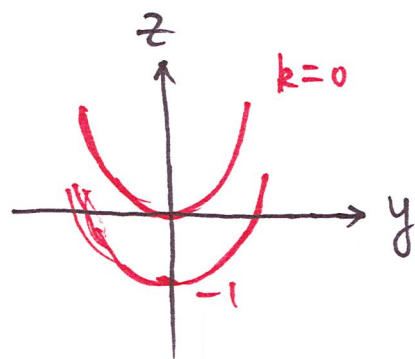
$$z = y^2 + 4k^2$$

$x=1$ $z = y^2 + 4$

Ex. 5 Sketch the surface $z = y^2 - x^2$

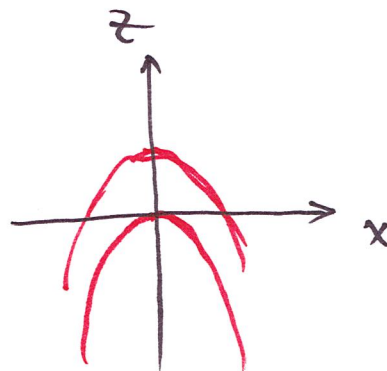
hyperbolic paraboloid

traces in $x=k$
 $z = y^2 - k^2$



$k = \pm 1 \quad z = y^2 - 1$

traces in $y=k$
 $z = k^2 - x^2$



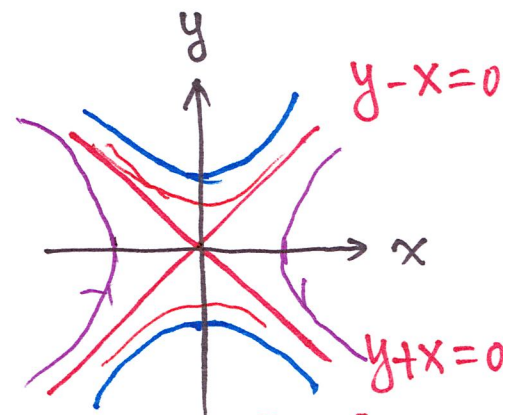
$k=0$ $z = -x^2$

$k=\pm 1$ $z = -x^2 + 1$

$k=1$ $y^2 - x^2 = 1$

$k=-1$ $y^2 - x^2 = -1$

traces in $z=k$
 $k = y^2 - x^2$



$k=0$ $y^2 - x^2 = 0$
 $\parallel$

$(y+x)(y-x)$

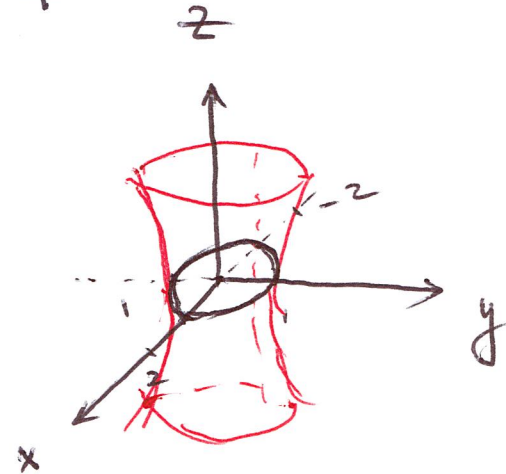
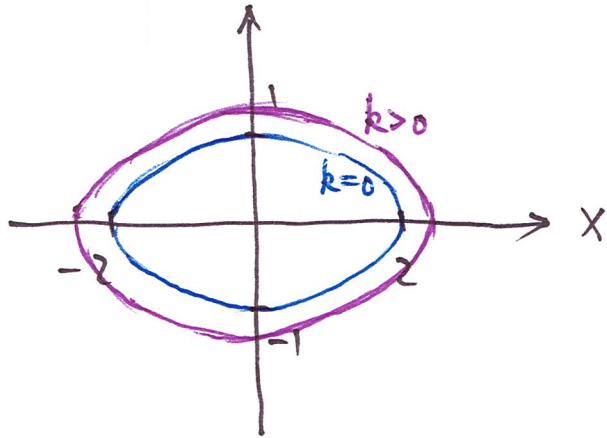
$y+x=0$ or $y-x=0$

Ex. 6 Sketch the surface $\frac{x^2}{4} + y^2 - \frac{z^2}{4} = 1$

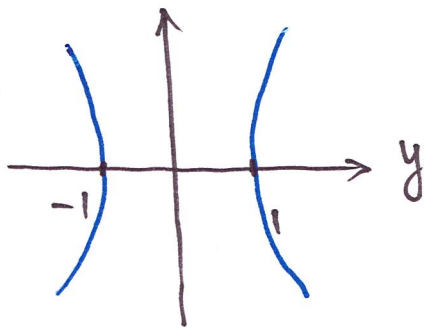
hyperboloid of one sheet

horizontal planes ($z = k$)

$$\frac{x^2}{4} + y^2 = 1 + \frac{k^2}{4}$$

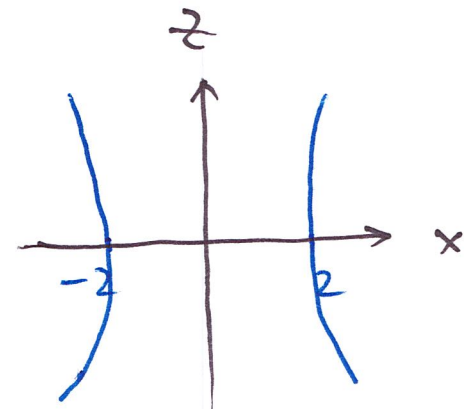


$x=0$ $y^2 - \frac{z^2}{4} = 1$



$y=0$

$$\frac{x^2}{4} - \frac{z^2}{4} = 1$$



Ex. 7 Identify and sketch the surface

$$4x^2 - y^2 + 2z^2 + 4 = 0$$

$$4x^2 - y^2 + 2z^2 = -4$$

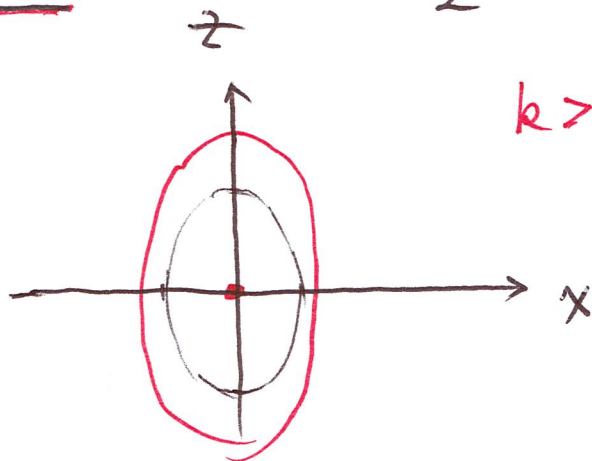
hyperboloid of two sheets

$$-x^2 + \frac{y^2}{4} - \frac{z^2}{2} = 1$$

$y = k$

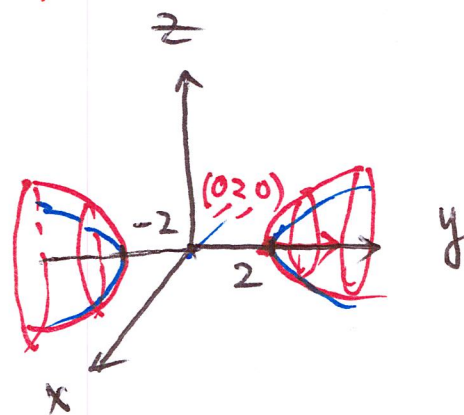
$$x^2 + \frac{z^2}{2} = \frac{k^2}{4} - 1 > 0$$

$$k > 2$$



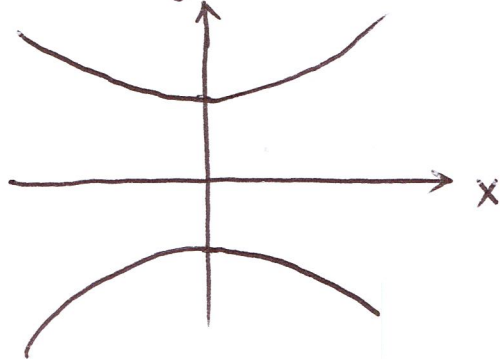
$$\frac{k^2}{4} - 1 \geq 0 \Rightarrow$$

$$\left\{ \begin{array}{l} k \geq 2 \\ \text{or } k \leq -2 \end{array} \right.$$



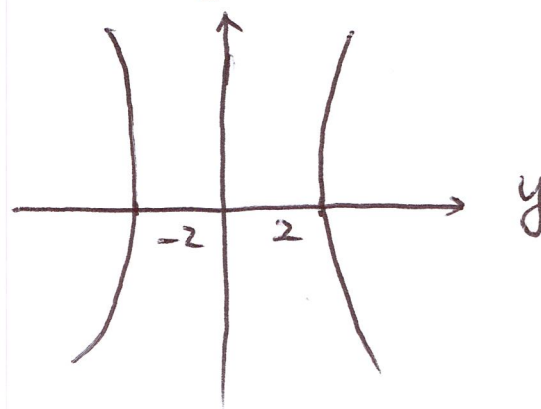
$z = 0$

$$-x^2 + \frac{y^2}{4} = 1$$



$x = 0$

$$\frac{y^2}{4} - \frac{z^2}{2} = 1$$



Ex. 8 Classify the quadric surface $x^2 + 2z^2 - 6x - y + 10 = 0$

$$0 = x^2 - 6x + 9 - 9 + 2z^2 - y + 10$$

$$= (x-3)^2 + 2z^2 - y + 1$$

$$(y-1) = (x-3)^2 + \frac{z^2}{\left(\frac{1}{\sqrt{2}}\right)^2}$$

$(3, 1, 0)$

